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MATRIX E_{ϕ}^{9}, CLASSIFICATION OF INTEGERS, PRIMORDIAL ALGEBRA.

MATRIZ E_{ϕ}^{9}, CLASIFICACIÓN DE ENTEROS, ALGEBRA PRIMORDIAL.

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Resumen

La Matriz E_{ϕ}^{9} es una herramienta que puede ser utilizada para la solución de problemas complejos, tomando su forma de reducir números y clasificarlos, con estas propiedades podemos crear grupos y observar sus propiedades para crear tablas, las cuales nos permiten ver el comportamiento de las operaciones en orden de crear predicciones de resultados, a través de un algebra especial que nos permite comprender mejor lo que ocurre en las operaciones.
Palabras clave:

Abstract

The matrix E_{ϕ}^{9} is a tool that can be used to order and solve complex problems, using its ability to reduce numbers and classify them, with these properties we can group and order all the integers, in this way creating classes prediction tables, which can give results that allow us to completely predict which are the numbers that can be a solution for operations and functions, the study of this ordering is through a special algebra that describes the behavior of numbers when being operated.

Keywords:

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1. Introduction

This article aims to study the possible applications that can be given to the Matrix E_{ϕ}^{9}, in addition to the use of new numbers, which will be called Primal, with these numbers we will proceed to create an algebra that allows us to predict the complex behavior of integers, thus making predictions with this new tool.

2. Introduction to the Matrix E_{ϕ}^{9}

The MatrixE_{ϕ}^{9} is the ordered set of integers ranging from positive to negative, which we will sort into sub-matrix one for negatives and one for positive ones.

Which for practical purposes of the article we will work on each one separately, however, it should not be forgotten that when we talk about MatrixE_{ϕ}^{9} we mean the whole positive and negative matrix.

2.1 A matrix$[V^{+}]_{ij}$ is positive if has the following characteristics:

- All its elements are positive integers.
- Every element is unique within the matrix there are no repeated elements
- The first element will be number 1 and will be located in the first row, the first column.
- In the first row, you should find the first nine positive integers, therefore 9 is the number of columns in the Matrix.
- The next row will consist of the following positive integers that come from 9, following the principle of the good ordering of the whole numbers. These read from left to right from 10 onwards. Likewise, the following rows of the matrix will be constituted, thus to infinity.
- Every integer belongs to a class ϕ^{k}, which is named after the "Primal Numbers" Z_{ϕ} (first numbers).
- Each integer belonging to a class ϕ^{k} shares a characteristic equal to that of the rest of the number within the same class.
- The Z_{ϕ} of each whole number can be found using the Digital Sum Function defined as "ϕ"

2.2. Definition of digital sum

Let v a decimal representation of an integer, the digital sum of v is a function $\phi|\mathbb{Z} \rightarrow \mathbb{Z}$ defined by

$$\phi_{(v)} \sum_{1 \le i \le n} a_i$$

where

$$v = \sum_{1 \le i \le n} a_i \, 10^i$$

Example:

if $v = 10 \; then \; \phi_{10} = 1 + 0 = 1$
if $v = 1034 \; then \; \phi_{1034} = 1 + 0 + 3 + 4 = 8$

And we'll have it over-understood that if we iterate infinitely the previous function, on a, $v \in \mathbb{Z}$ we will get a cycle that will lead to single-digit numbers, contained in the following set:

$$D = \{d | -9 \le d \le 9 |\}$$

$D \in \mathbb{Z}$**:**(D, It will be the set of fixed points in the k-Times-iterated sequence.) Since the set of fixed points of the sequence obtained from the k-TIMES iteration the function "ϕ" above v, will be D, then you will have to:

$\phi: \mathbb{Z} \to \mathbb{Z}$,

$$\phi(v) = \begin{cases} id_{\mathbb{Z}} \\ (-1) \sum_{i=0}^{n} |a_i| \\ \sum_{i=0}^{n} a_i \end{cases} \begin{cases} if \; v \in D \\ if \; v \in \; \mathbb{Z}^- \; D \\ if \; v \; \in \; \mathbb{Z}^+ - D \end{cases}$$

Where the elements of will be denoted as v_m, $m \in N_0$
Now regarding the cases where we will have a "v" such that $\phi(v) \neq d$, where $d \in D$, we should keep iterating m-iteration to "ϕ" on v, with the aim of the Orbv (Orbit of v) reach a value "d" (A value of the set of fixed points).
So, next, we'll introduce a notation for the m-iteration of v to ϕ.

(1). let ϕ^m The m-Esima iteration of the ϕ over v, where $m \in \mathbb{N}_0$:
$m \in N_0$ / ϕ^0= $id\mathbb{Z}$, where $id\mathbb{z}$ will be the identity function in $\mathbb{z}$.

(2). Then $\phi^m = (\phi \circ \phi \circ \phi \circ \ldots \circ \phi)(v)$.

The following Abelian property of the functions needs to be taken into account to continue the possible definitions of ϕ^m

Abelian property of functions

(3). Be: ϕ^m or ϕ^n= ϕ^n or ϕ^m =$\phi^n\big(\phi^m\,(v)\big) = \phi^{n+m}$, $\forall m, n \in \mathbb{N}_0$

So from this property, we can argue the following

(4). ϕ^{m+1}=ϕ or ϕ^m

So we'll also have to

(5). ϕ=$\phi(\phi^{m-1}(v))$

Thus and from (5) we can make the following definitions:

- For all integers such that $v = v_m$, where m will indicate the number of the function "ϕ" to be applied over v.
  You have to our "v" will be $v = v_0$

- Then we can define more specifically to (5) as follows:
  $\phi^m(v_m) = v_{m+1}$ for $m \geq 0$, and being"ϕ^m" a Picard function, then
  $\phi^0(v) = v \therefore \phi^0(v_0) = v_1$.
  $\phi^0 = id\mathbb{Z}$, for $m = 0$. (6)

If v is a fixed point of the set D, then there will be a d element, of the set of fixed points D such that $\phi^0(v) = d$.

The concept of Marrero Condition is then introduced.

2.3. Condition of Marrero (ϕ_k^M)

Given the function $\phi^m(v_m)$ for an integer $m \geq 1$, we say that it satisfies the condition of Marrero if it meets the following expression:

$\exists!\, d \in D \;/\; \phi^m(v_m) - d = 0$

(ϕ_k^M). The Condition of Marrero acts as an iterative stop criterion since when the condition is met, this will indicate that this function cannot continue to interact, due to the only digit that makes up "v_m" resulting from m-Times "ϕ" over v.

(6) Let v an integer. Then $\phi^0 = d$ if and only if $v = d$, for an integer of D.

2.4. Proofs:

Case 1: ($v \in \mathbb{Z}^+$)

- Let $v \neq d$, for an integer $d \in D$, then d will be an integer consisting of a digit, such that$d = b_0 * 10^0 = b_0$ or $d = -b_0 - 10^0 = -b_0$, where the terms "b" will be the constituent digits of d.

  Then, when you have $v \neq d$, you'll have to like $v \in \mathbb{z}+$, then $v = a_n 10^n + a_{n-1} * 10^{n-1} + a_{n-2} * 10^{n-2} \ldots + a_2 * 10^2 + a_1 * 10 + a_0 * 10^0 \neq \pm b_0 = d$. In addition to the above, we have to by definition $\phi^0(v) = id\mathbb{Z}$, so you also have to $\phi^0(v) = v$. therefore $v \neq d$, then finally $\phi^0(v) = v \neq d$.

- For a $v \in \mathbb{Z}^+$, assume that $\phi^0(v) \neq d$ and as by definition, we have the fact of $\phi^0(v) = id\mathbb{z}$; then we can say that for a positive integer of the form $v = a_n 10^n + a_{n-1} * 10^{n-1} + a_{n-2} * 10^{n-2} \ldots + a_2 * 10^2 + a_1 * 10 + a_0 * 10^0$, it is fulfilled that $\phi^0(v) = v$. Then, like D, we have d will be an integer formed by a digit. In other words, $d = b_0$ o $d = -b_0 * 10^0$. therefore, and from the latter, if $\phi^0(v) = v$ and as $\phi^0(V) = v \neq d$, we can conclude that $v \neq d$.

Case 2: ($v \in \mathbb{z}^-$)

- Let a $v \neq d$, for an integer $d \in D$, then is the case d will be an integer made up of a digit, such as $d = b_0$. Or in the case of being negative $d = -b_0$.
  Then and being $v \neq d$, you'll have to $v \in \mathbb{Z}^-$, then $v = -(a_n 10^n + a_{n-1} * 10^{n-1} + a_{n-2} * 10^{n-2} \ldots + a_2 * 10^2 + a_1 * 10 + a_0 * 10^0) \neq \pm b_0 = d$.

In addition, we have to by definition $\phi^0(v) = id\mathbb{Z}$, then $\phi^0(v) = v$. Therefore, as $v \neq d$, then we finally have $\phi^0(v) \neq d$.

- For a $v \in \mathbb{Z}^-$ let's assume that $\phi^0(v) \neq d$ and as by definition, we have the fact that $\phi^0(v) = id\mathbb{Z}$; then we can say that for a v of the form $v = -(a_n 10^n + a_{n-1} * 10^{n-1} + a_{n-2} * 10^{n-2} \ldots + a_2 * 10^2 + a_1 * 10 + a_0 * 10^0)$, it is fulfilled that $\phi^0(v) = v$. Then and as $d \in D$, we have d will be and integer formed by a digit; therefore $d = \pm b_0 * 10^0$, if $\phi^0(v) = v$, we can conclude from the fact that $\phi^0(v) \neq d$, that for an integer v negative.

Case 3:$v = 0$

- Suppose that $v \neq d$, for an integer $d \in D$, then as d is a digit number, d will be $d = \pm b_0 * 10^0 = \pm b_0$. From the fact that v is also a digit number, being $v = 0$; we can then affirm that $v = a_0 * 10^0 = a_0$, where $a_0 = 0$. So if $v \neq d$, then $v = a_0 \neq b_0 = d$. Then we have to by definition $\phi^0(v) = id\mathbb{Z}$, then $\phi^0(v) = v$ therefore, as $v \neq d$, then we can end by stating that $\phi^0(v) \neq d$.

- For a $v = 0$, assume that $\phi^0(v) \neq d$ and as by definition we have to be $\phi^0(v) = id\mathbb{Z}$; then we can say that for a $v = 0$ and the form $V = a_0$, it is true that $\phi^0(v) = v$. In addition to the above, such as $d \in D$, then also d is a digit number such that $d = \pm b_0$. Therefore, and from the latter, if $\phi^0(v) = v$, and having found that $\phi^0(v) \neq d$, we can conclude that $v \neq d$.

- Depending on the values obtained by the m-sub-index, the orbit of $\phi^0(v)$, will behave as follows:

For an $m \geq 0$, $\quad \phi^0(v_0) = v_1$

$$\phi^0(v_1) = v_2$$
$$\phi^0(v_2) = v_3$$
$$\phi^0(v_3) = v_4$$
$$\vdots$$

$$\phi^m(v_m) = v_{m+1}.$$

Next, we'll introduce the main features of the function ϕ

So be it $\phi^m : \mathbb{Z} \to \mathbb{Z}$.

2.5. Lemma

Before introducing the DEI Lemma, it is necessary to take into account these characteristics concerning the digital sum (ϕ) and the Condition of Marrero ($\boldsymbol{\phi_k^M}$)

1) As defined above, $\boldsymbol{v_0 = v_1}$ or better known as our initial integer number. So for the following cases where iterate m-Times ϕ over that a $\boldsymbol{v_0 = v_1}$, we have the following:

Let

$$\begin{cases} \boldsymbol{v_1 = a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0} \\ \boldsymbol{v_2 = b_n b_{n-2} b_{n-3} \dots b_2 b_1 b_0} \\ \boldsymbol{v_k = x_n x_{n-1} x_{n-2} \dots x_2 x_1 x_0} \end{cases}$$

Then

$$\phi^1_{(v_1)} = \sum_{i=0}^{n} a_i = v_2, \qquad \textit{where the } a_i \textit{ are the constituent digits of } v_1$$

$$\phi^2_{(v_2)} = \sum_{i=0}^{n} b_i = v_3, \qquad \textit{where the } b_i \textit{ are the constituent digits of } v_2$$

.

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$$\phi^m_{(v_m)} = \sum_{i=0}^{n} x_i = v_{m+1}, \qquad \textit{where the } x_i \textit{ are the constituent digits of } v_m$$

Therefore, if we have the case that the$\phi^m_{(v_m)} - d = 0$ will have $v_m + 1 = v_m$ and so the $v_m + 1 \in D$.

When we have a function that complies with $(\boldsymbol{\phi_k^M})$ such as $\phi^M_{(v)} = v_M$, then $\phi^M_{(v)} = z_0 * 10^0$, where z_0 is the constituent digit number of v_M.

Taking into account the above we can enter the Lemma that reads as follows:

2.6. Lemma DEI.

For any non-negative v_1, for which the following is met:

if $\phi^1_{(v_1)} = v_2$ such as $v_2 \in D$, then $\phi^1_{(v_1)} = rem(v_{1,9})$.†

2.7. Proof:

let

$$\phi^1_{(v_1)} = \sum_{i=0}^{n} a_i$$

then we know

$$\sum_{i=0}^{n} a_i = \boldsymbol{a_0} + \boldsymbol{a_1} + \boldsymbol{a_2} + \cdots + \boldsymbol{a_{n-2}} + \boldsymbol{a_{n-1}} + \boldsymbol{a_n}$$

Also, if

$$\sum_{i=0}^{n} a_i = v_2$$

So the $v_2 \in D$, we have that if we assign by b_i to the constituent digits of the integer v_2, we will get by definition that $(\boldsymbol{b_n b_{n-2} b_{n-3}} \ldots \boldsymbol{b_2 b_1 b_0}) = 0 \therefore b_0 - 10$.

then as the division theorem states (Introducción a la Teoría de Números. "Walter Mora F". p.15.) [1]; we have $v_1 = 9c + rem(v_1, 9)_†$, from where $c \in \mathbb{N}_0$.

therefore $rem(v_1, 9) = v_1 - 9c$, and like any non-negative integer can be written as$v = 10^n a_n + 10^{n-1} a_{n-1} + \cdots + 10^2 a_2 + 10^1 a_1 + 10^0 a_0$, from where it is.

$$v = 9\left[a_1 + 11a_2 + \cdots + \left(\frac{10^{n-2}-1}{9}\right)a_{n-2} + \left(\frac{10^{n-1}-1}{9}\right)a_{n-1} + \left(\frac{10^n-1}{9}\right)a_n\right] + (\boldsymbol{a_0} + \boldsymbol{a_1} + \boldsymbol{a_2} + \cdots + \boldsymbol{a_{n-2}} + \boldsymbol{a_{n-1}} + \boldsymbol{a_n})$$

therefore

$$\left[a_1 + 11a_2 + \cdots + \left(\frac{10^{n-2}-1}{9}\right)a_{n-2} + \left(\frac{10^{n-1}-1}{9}\right)a_{n-1} + \left(\frac{10^n-1}{9}\right)a_n\right] \in N_0$$

So we can say that

$$\left[a_1 + 11a_2 + \cdots + \left(\frac{10^{n-2}-1}{9}\right)a_{n-2} + \left(\frac{10^{n-1}-1}{9}\right)a_{n-1} + \left(\frac{10^n-1}{9}\right)a_n\right] = c$$

so $c \in N_0$. We see that

$$v = 9c + (\boldsymbol{a_0} + \boldsymbol{a_1} + \boldsymbol{a_2} + \cdots + \boldsymbol{a_{n-2}} + \boldsymbol{a_{n-1}} + \boldsymbol{a_n})$$

$(*)$ And as we could see in the division theorem [1]

$0 \leq rem(v_1, 9) < |9| \therefore si\ \phi^1_{(v1)} = v_2$ Tal que $v_2 \notin D$

So this means that being

$v_2 = \boldsymbol{b_n b_{n-2} b_{n-3} \dots b_2 b_1 b_0}$ you have

$(\boldsymbol{b_n b_{n-2} b_{n-3} \dots b_2 b_1 b_0}) \neq \boldsymbol{0},$ therefore $\phi^1_{(v1)} > 9. (*)$.

Continuing, when we run into the following:

$$(\boldsymbol{a_0} + \boldsymbol{a_1} + \boldsymbol{a_2} + \cdots + \boldsymbol{a_{n-2}} + \boldsymbol{a_{n-1}} + \boldsymbol{a_n}) = \boldsymbol{v_1} - \boldsymbol{9c}.$$

From the above and by definition the digital sum of non-negative integers we have to

$$\phi^1_{(v_1)} = v_1 - 9c$$

And as we know, in this case, $\phi^1_{(v1)} = v_2$ where $v_2 \in D$ then:

$$(\boldsymbol{b_n}, \boldsymbol{b_{n-1} b_{n-2} \dots b_2 b_1}) = \boldsymbol{0}$$

So from the latter, we can conclude that $v_1 \in N_0$; it will be fulfilled that :

$0 \leq \phi^1_{(v1)} < |9|$.

$(**)$It is also important to note that the initial numbers of the form $v_1 = 9c$, for some $c \in \mathbb{N}_0$, which tells us that $9|_{v1} \therefore v_1 \equiv 0(mod 9)$

In the same way as the v_2 result of $\phi^1_{(v1)}$ is such that $v_2 \in D$, and on the other hand as the divisibility criterion demonstrates, if $v_1 \equiv 0(mod9)$, then:

$$v_1 \equiv a_0 + a_1 + a_2 + \cdots + a_{n-2} + a_{n-1} + a_n\ (mod9)$$

Therefore

$$v_1 \equiv \phi^1_{(v1)} \equiv 0\ (mod9).$$

Starting whit $v_1 \equiv \phi^1_{(v1)} \equiv 0\ (mod9)$ and coupled with the fact that$v_2 \in D$, we can verify that.

$\sum_{i=0}^{n} a_i = v_2 = 9$

We can use by convention that $\phi^1_{(v1)} = 0$, because as $9|_{v1} \rightarrow rem(v_1, 9) = 0. (**)$.

In conclusion, taking into account a $(*)$ and $(**)$ and the equation $\phi^1_{(v1)} = v_1 - 9c$, we have proof that:

$$\forall_{v_1} \in \mathbb{N}_0 / v_2 \in D, \text{by } \phi^1_{(v1)} = v_2. \text{ then } \phi^1_{(v1)} = rem(v_1, 9).$$

Therefore we can end by stating that under these conditions $v_1 = 9c + \phi^1_{(v1)}$, for a $0 \leq \phi^1_{(v1)} < |9|$ if we use the above convention.

3. Matrix construction method

Let $Mmxn(\mathbb{Z}^+)$ the set of all matrices with m-rows and n-columns that have $\mathbb{Z}^+$. So if $m, n \in \mathbb{Z}^+$ between V^+ it will be an matrix of elements of the shape:

$$V^+ = \begin{pmatrix} v_{11} & \cdots & v_{19} \\ \vdots & \dots & \vdots \\ v_{m1} & \cdots & v_{m9} \end{pmatrix}$$

Where the $m\ n - uplas$: $(v_{11} \dots v_{19}) \dots (v_{m1} \dots v_{m9})$ are the rows in the matrix V^+.
By the other parts $n\ m - uplas$ de V^+:

$$\begin{pmatrix} v_{11} \\ \vdots \\ v_{m1} \end{pmatrix}, \dots, \begin{pmatrix} v_{19} \\ \vdots \\ v_{m9} \end{pmatrix}$$

The columns of V^+

the $v_{i,j}$ element in the V^+ matrix appears in the row i and column j.
Integer matrix is defined as:

Let $V^+ \in M_{mxn}(\mathbb{Z}^+)$ such that $[V^+]_{ij}$ be an integer when:

$$v_{ij} = 9(i-1) + J$$

3.1. Matrix row calculation method $[V^+]_{ij}$

Definition:

Let the entire matrix $[V^+]_{ij}$; if we have that each element of $[V^+]_{ij}$

Is represented by v, then the Primal Number that is responsible for representing the row in which v is located in the 9-column matrix is given by:

Table 1.

$$Matrix\ E_{\phi}^{9}\ in\ the\ \mathbb{Z}^+ = [V^+]$$

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

The Primal Classes that exist in the matrix $[V^+]$ are 9, classes: 1, 2,3…9.

Which are given by the n-columns that make up the matrix, $[V^+]_{ij}$.

In which each number follows the principle of good sorting of numbers. Each class will be identified as v_{ϕ} where the class representative number is ϕ.

This creates a dimension in which you can carry numbers and operations of the common algebra, and observe the iterations that led to why those results also allow for an easier understanding of the behavior of the different types of numbers that exist, as long as they are integers.

Each number has an ordered pair of coordinates that are (i, j), from where i is the horizontal coordinate (row position); j is the vertical coordinate (column position)

Having position i, j using the equation:

$$v_{ij} = 9(i-1) + j$$

$$j_v = 9(i-1) - v$$

$$i_v = \begin{cases} \left\lfloor \dfrac{v}{9} + 1 \right\rfloor & if\ v \neq 9c, for\ \ c \in \mathbb{Z}^+ \\ \dfrac{v}{9}\ if\ v = 9c, & for\ \ c \in \mathbb{Z}^+ \end{cases}$$

As a verification method, you can use the following equation, the result must be i obtained as an integer.

$$i_v = \frac{v-j}{9} + 1$$

The i-row demonstration is found in a general way for the natural MN matrix and its i-row theorem. (RAMIREZ LOPEZ J y GORROSTOLA NADAD J., 2015," Suma sucesiva", p.51. Available in Journals and publications of the University of the Atlantic at:http://investigaciones.uniatlantico.edu.co/revistas/index.php/MATUA)[2]

3.2. An Matrix $[V^-]_{ij}$ is negative if it has the following characteristics:

- All elements are negative integers.
- Any element is unique within the matrix (no number is repeated).
- The first element will be the number -1 and will be located in the first row, the first column.
- In the first row, you must find the first nine negative integers from highest to lowest, therefore 9 is the number of columns in the Matrix. (No. 9).
- The second row will consist of the next nine integers that come to the number -9, from highest to lowest. These will be read from left to right from -10 onwards. Similarly, the following rows of the Negative Whole matrix will be constructed $[V^-]$**.**
- Every integer belongs to a class of ϕ^k, which is named after the "Primal Numbers" Z_ϕ (first numbers).
- Each integer that belongs to a class ϕ^k shares a characteristic equal to that of the rest of the number within the same class.
- The Z_ϕ of each integer can be found using the Digital Sum Function defined as "ϕ"

Definition:
Let $V^- \in M_{mxn}(\mathbb{Z}^-)$ such that $[V^-]_{ij}$ be entirely negative when:

$$v = -9(i-1) + j$$

3.3. Matrix row calculation method $[V^-]_{ij}$

As whit $[V^{+}]_{ij}$, i represents the i-row position of the v element in $[V^{-}]_{ij}$. Then i-row will come given as follows:

$$i\begin{cases}\left\lfloor -\frac{v}{9}+1\right\rfloor, if\ v \neq 9c, & for\ \ c \in \mathbb{Z}^{-} \\ -\frac{v}{9}, \quad if\ v = 9c, & for\ \ c \in \ \mathbb{Z}^{-}\end{cases}$$

3.4. Proof:

By definition of the Negative Whole Matrix, it is known that $v = -9(i-1)+j$. now we have to:

$$v - j = -\left(\frac{v-j}{9}\right)$$

$$i - 1 = -\left(\frac{v-j}{9}\right)$$

$$i = \frac{-v+j}{9} + 1$$

$$i = \left(\frac{-v+j+9}{9}\right)$$

$$i - \frac{j}{9} = \left(\frac{-v+9}{9}\right)$$

therefore

$i - \frac{j}{9} \in \mathbb{N}, as\ i, j\ \ \in \mathbb{Z}.$

Then, and from the last

$$i \leq i - \frac{j}{9} < i+1$$

Where

$i \leq \left(\frac{-v+9}{9}\right) < i+1$

Finally, by definition of the floor function (“Introducción a la Teoría de Números”. Walter Mora F. p.6.) [3]

We have to:

$$\left\lfloor -\frac{v}{9}+1 \right\rfloor$$

The latter occurs when v is not a multiple of 9, which is also the number of columns in the matrix $[V^{-}]_{ij}$. On the other hand, when v is a multiple of 9, we have to $j = -9$, since j is the position in the column. Now we check for case $j = -9$. Then:

$i - \frac{j}{9} = \left(\frac{-v+9}{9}\right)$

We have

$i + 1 = \left(-\frac{v}{9}+1\right)$

$i = \left(-\frac{v}{9}+1\right)+1$

$i = \left(-\frac{v}{9}\right)$

(substituting $j = -9$)

3.6. Matrix column calculation method $[V^{-}]_{ij}$

Since v represents any element of $[V^{-}]_{ij}$, we have to

$$j = \begin{cases} v + 9\left(\left\lfloor -\frac{v}{9} \right\rfloor\right), & if\ \ v \neq 9c, for\ a\ c \in \mathbb{Z}^{-} \\ -9, & if\ v = 9c, for\ a\ c \in \mathbb{Z}^{-} \end{cases}$$

3.7. Proof:

As we know $v = -9(i-1) + j$ by definition of Negative Whole Matrix, then we can see the following

$j = v + 9(i-1)$

$j = v + 9\left(\left\lfloor -\frac{v}{9} + 1\right\rfloor - 1\right)$

$j = v + 9\left(\left\lfloor -\frac{v}{9}\right\rfloor + \lfloor 1\rfloor - 1\right)$

$j = v + 9\left(\left\lfloor -\frac{v}{9}\right\rfloor\right)$

Therefore, we can verify that when $v = 9c, for\ a\ c \in \mathbb{Z}^-$, you get a $j = 0$, y by convention $(**)$ mentioned in the lemma DEI that $j = -9$.

4. Theorem $\boldsymbol{\phi^{M}_{(v)}}$

If $\phi^{m}_{(v_m)} = rem(v,9)/v \in \mathbb{Z}^+$, then $\phi^{m}_{(v_m)}$ is a function $\phi^{M}_{(v)}$

4.1. Proof:

Let

$v_k = \boldsymbol{x_n x_{n-1} x_{n-2} \ldots x_2 x_1 x_0}$

Then

$$\phi^{m}_{(v_m)} = \left(\sum_{i=0}^{n} x_i\right)$$

Then we can see by the Lemma DEI that $\phi^{1}_{(v1)} = -9c$, that also $rem(v,9)v - 9c$ therefore if $\phi^{1}_{(v1)} = rem(v,9)$,
Then the possible value orbit of $\phi^{1}_{(v1)}$ will be $0 \leq \phi^{1}_{(v1)} < |9|$, from where we can infer that

$\sum_{i=0}^{n} v_{m+1}$such that $v_{m+1} \in D$.

Therefore if $v_{m+1} \in D$, who is designated the form

$v_{m+1} = z_n z_{n-1} z_{n-2} \dots z_2 z_1 z_0$, where from $\phi^{m}_{(v_m)} = rem\ (v,9)$

It will have to $(z_n z_{n-1} z_{n-2} \dots z_2 z_1 z_0) = 0 \therefore v_{m+1} = 10^0 z_0$

And the latter affirms

$\exists!\ d \in D\ such\ that\ v_{m+1} - d = 0$, which states that $v_{m+1} \in \phi^{M}_{(v)}$,

In addition, by the theorem of the division of $\mathbb{z}$, which:

$$\forall_v, c \in \mathbb{N}_0, v = 9c + \phi^{M}_{(v)}.$$

4.3. Theorem $\phi^{M}_{(v)}$ applied

For any integer $v_m \in N_0$, for which $\phi^{m}_{(v_m)} = rem\ (v,9)$.

There is a single $\phi^{m}_{(v_m)} \in N_0$ such as $v = v_m, v = 9\lfloor {}^{v}/_{9} \rfloor + \phi^{M}_{(v)}$ whit $0 \leq \phi^{M}_{(v)} < 9$.

4.4. proof:

As it is understood that the theorem of division exists in

$c \in N\ such\ as\ 9c \leq v < 9(c+1)$, then by definition of the floor function We can say that $v \in R$ and hence

$$c = \lfloor {}^{v}/_{9} \rfloor \therefore \phi^{M}_{(v)} = v - 9\lfloor {}^{v}/_{9} \rfloor$$

Uniqueness: Assume that there is a $[\phi^{M}_{(v)}]', c' \in N_0\ such\ as\ v = 9c' + [\phi^{M}_{(v)}]'$ whit

$0 \leq [\phi^{M}_{(v)}]' < |9|$.

Next, we're going to assume that $\phi^{M}_{(v)} \neq [\phi^{M}_{(v)}]'$, what besides $\phi^{M}_{(v)} > [\phi^{M}_{(v)}]'$.

$$\begin{cases} v = 9c' + [\phi'_{(v)}]' \\ \ \ v = 9c + \phi'_{(v)} \end{cases}$$

Now matching you love equations we get the following

$9c' + [\phi'_{(v)}]' - (9c + \phi^{1}_{(v)}) = 0$

$9c' + [\phi'_{(v)}] - 9c - \phi^{1}_{(v)} = 0$

$9(c' - c) = \phi'_{(v)} - [\phi'_{(v)}]' \rightarrow 9|\left(\phi'_{(v)} - [\phi'_{(v)}]'\right)$

Therefore that $9|\left(\phi'_{(v)} - [\phi'_{(v)}]'\right)$, we have $\phi'_{(v)} - [\phi'_{(v)}]' \geq 9$

Also we know $0 < \phi'_{(v)} - [\phi'_{(v)}]' \leq \phi'_{(v)} < 9$; which is absurd.

So by reducing absurdity, we conclude that:

$$\phi'_{(v)} = [\phi'_{(v)}]' \therefore 9(c' - c) = \phi'_{(v)} - [\phi'_{(v)}]' = 0 \rightarrow c' = c$$

5. Elements that make up the Matrix E^{9}_{ϕ}

As noted above, the matrix E^{9}_{ϕ} is a set of two matrixes that classify and sort integers in such a way that we can observe the patterns that each number has, as well as being able to build predictions when solving a complex problem.

But before starting with that, it is necessary to understand the elements that compose it to establish a dimensional order where you can work.

The elements of the matrix E^{9}_{ϕ} will then be exposed in a hierarchical order from lowest to highest, starting from their simplest element to their most complex components.

- Integer number (v)
- Primal number (Z_{ϕ})
- Classes (ϕ^{k})
- Macro Primal (ζ)

5.1. Notation of Regular Numbers in the Matrix

For the numbers that make up the matrix will be v_{ϕ}, where $v \in \mathbb{Z}$ and ϕ is the result of computing $\phi^{M}_{(v)}$. The numbers will be organized whit respect to the $j - column$ position that each element has $v \in [V^{+}]_{ij}$ and $v \in [V^{-}]_{ij}$

5.2. example:

let

$$v = 12$$

We have that

$$j = 12 - 9\left\lfloor\frac{12}{9}\right\rfloor = 12 - 9 = 3$$

Therefore, if $j = 3$ would remain the notation v_ϕ as 12_3 since it $\phi_{12}^M = 3$.

5.3. Primal Numbers

Primal numbers are the first-order numbers that exist within the matrix. That is, for each joint class that exists in the matrix, there will be a symmetrical convergence number which we will call as "Primal Number" (Z_ϕ).

Its value represents the congruence that exists between the s $j - coordinates$ and each number in that same column.

Let.

$$j = Z_\phi$$

Which means that

$$\forall\, v \in j\, \exists\, Z_\phi \,/\; Z_\phi \equiv j\ (mod\ 9)$$

With these, we define the value of a Primal Number, which will be used to classify the numbers.

An example of notation of a primal number

$$42 = 6_\phi$$

where 6_ϕ is the primal number of 42, so the primal number will be the representative number

5.4. Classes ϕ^k

Primal Classes are defined by the Primal Number that characterizes a j-column.

Unlike Primal Numbers, we will refer to Classes, like the set that exists within the matrix.

For such classification, we will use the notation ϕ^k where the value of "k" will be the value of Z_ϕ

Classify the numbers in this way:

$$k \equiv Z_\phi$$

Where ϕ^k is the name of the class set and Z_ϕ is the numeric value of the class set.

5.5. We define:

for an integer, $v \in [V^{+}]_{ij}$ or $v \in [V^{-}]_{ij}$, the Primal Class of this element in the matrix E_{ϕ}^{9} to which it belongs will be given by the j that owns the v.
The class will be denoted as $\phi^{k} = v_{\phi}$ where $\phi = \phi_{(v)}^{M} = Z_{\phi}$
At the en we can define ϕ^{k} as:

$$\phi^{k} = \{v \in \mathbb{Z} \,|Z_{\phi} \equiv v(mod\ 9)\}$$

5.6. Method of calculating classes
The Classes and Primal Number can be calculated using any function $\phi_{(v)}^{M}$ this is given by the definition given above, where any function that can define a j will also tell us the class of that number.

$$\phi_{(v)}^{M} = \phi^{k} \ \rightarrow \ \phi_{(v)}^{M} = Z_{\phi}$$

5.6.1. Proof:

By definition of Primal Classes, We have $Z_{\phi} = v_{\phi}$,
Then if:

$$\phi^{k} = \begin{cases} v - 9\left(\left\lfloor \frac{v}{9} \right\rfloor\right), & si\ v \neq 9c, for\ an\ c \in \mathbb{Z}^{+} \\ 9, \quad si\ v = 9c, & for\ an\ c \in \mathbb{Z}^{+} \end{cases}$$

We see

$$v - 9\left(\left\lfloor \frac{v}{9} \right\rfloor\right) = v_{\phi}$$

Then for Theorema of $\phi_{(v)}^{M}$ applied, we can check

$\phi_{(v)}^{M} = v - 9\left(\left\lfloor \frac{v}{9} \right\rfloor\right)$, whit $0 \leq \phi_{(v)}^{M} < 9$.

Now

$$\phi_{(v)}^{M} = v_{\phi} \therefore \phi_{(v)}^{M} = Z_{\phi}$$

5.7. Primal Class Properties.

- Equality Property Rest Class:
  like $\phi^{M}_{(v)} = v_{\phi}$, of the theorem $\phi^{M}_{(v)}$ we get the following equality:
  $rem(v, 9) = v_{\phi}$.
- Property of congruence

$$v \equiv \phi^{M}_{(v)} \ (mod\ 9)$$

- Property of Super Symmetry

$$v \equiv \phi^{M}_{(v)} \equiv \phi^{k} \equiv Z_{\phi} \ (mod\ 9)$$

5.8. Set Macro Primal(ζ_{ϕ})

The Macro Primal set is the set that encloses all the Primal Numbers that exist. According to the supersymmetry properties exposed in the previous example, it says.

$$\phi^{M}_{(v)} \equiv \phi^{k} \equiv Z_{\phi} \ (mod\ 9)$$

We have that the possible values for the Z_{ϕ} that contain both matrix $[V^{+}]_{ij}$ and $[V^{-}]_{ij}$ can only belong to the set of fixed values D.

$$\{\zeta_{\phi}\} = \{\phi^{k} | \phi^{k} \in \mathbb{Z}\}$$

5.9. Graphical representation of the matrix hierarchy E^{9}_{ϕ}

$$\forall\, E^{9}_{\phi} \ \exists \ \{\zeta_{\phi}\} = \begin{cases} +\{\phi^{k}\} \equiv Z^{+}_{\phi} \in [V^{+}]_{ij} \in \mathbb{Z}^{+} \\ \emptyset \\ -\{\phi^{k}\} \equiv Z^{-}_{\phi} \in [V^{-}]_{ij} \in \mathbb{Z}^{-} \end{cases}$$

This is just a representation of how all the elements that exist in the Matrix Theorem are sorted, knowing exactly which elements and where you are located is what helps facilitate the use of it. For each level, we will use different notations so we will make a table of terms and examples.

Table 2.

Element	Notation	Example
Numbers	v_{ϕ}	54_{9}
Classes	ϕ^{k}	ϕ^{9}
Primal Numbers	Z_{ϕ}	9_{ϕ}

6. *Algebra Primordial*

After understanding the fact that we can reduce any integer to a single-digit number, and knowing that there are only eighteen of them, nine positives and nine negatives, which we then group into columns of a matrix using that number to which they collapse when a digital sum is applied, we can begin to understand that there is not only one pattern if not intrinsic behavior in all the numbers that can help us solve problems more complex.

The numbers that we obtain from the digital sum, the numbers of the first order those of a single digit we have called them as primordial or Primal numbers, are based on these numbers that we will work the theorem.

- Acuña's theorem

Any behavior that has a primal number with an algebraic operation, either with the same or with a different primal number, will be the same behavior that all the numbers in the class set that represent that or those primal numbers have.

$$\forall\, Z_{\phi a} \ \odot \ Z_{\phi b} = \ Z_{\phi c} \cong \ v_a \odot v_b = v_c$$

This will help us understand how integers move within matrices, allowing us to observe symmetry within each operation, so when we operate with integers that are inside the matrix we must take into account that what is happening is that we are moving from one value to another within the matrix. Understanding these algebraic behaviors will help us understand how whole numbers work when operated.

The operations of subtraction, addition, and multiplication, are the ones that we will study next, logically this seems to describe a group, ergo if the existence of a group is demonstrated, the veracity of a part of the theorem is demonstrated.

6.1. Gruop Macro Primal $G\{\zeta_\phi\}$

If the primal numbers that have group properties must meet the conditions defined by a group:

- They exist as group $\{\zeta_\phi\}$

The set $\{\zeta_\phi\}$ is defined in 5.8. It is the existence of the eighteen representative elements that make up both sub-matrices.

$$\{\zeta_\phi\} = \left\{\phi^k \mid k \in D - \{0\}\right\}$$

- Exist an opposite element:

Each primal number has an opposite element, for the positive Primals evidently its opposite is the negative ones.

$$\forall\, Z_\phi \in \{\zeta_\phi\}, \exists\, Z_{-\phi} / Z_\phi + Z_{-\phi} = i$$

- Exist an element of identity: "i"

We define i such that:

The Primal numbers that possess the behaviors of 0 are ϕ^{-9} and ϕ^9 representing all the elements of the ninth column 9 positive and the negative -9, this is due to congruency with module 9, which tells us that any element that is multiple of 9 has a residue 0.

$$\phi^{-9} \cong \phi^9 \cong 0\ (mod\ 9) \to 9 | \phi^{-9} - \phi^9.$$

This does not mean that the class of 0 exists, zero is a unique number to which it cannot be reached by adding the parts of an integer greater or less than it, so its category would be of a special number, that to differentiate it from that congruence and give a more appropriate meaning to what it represents we will use the symbol of $\emptyset$.

$$\nexists \boldsymbol{v} \in \phi^0 \rightarrow \phi^0 = \emptyset$$

In the case of multiplication, the neutral element would pass to class ϕ^1 this is also the product of the congruence of the ϕ^1 $(mod\ 9)$.

6.2. Primal Operations.

Taking into account the above, we will start with the realization of examples of the operations, in addition to making a comparison with the integers they represent, in this way we can confirm what the Theorem of Acuña says.

6.2.1. Primal Sum.

Let

$$Z_{\phi a} + Z_{\phi b} = Z_{\phi c}$$

This operation consists of adding several values in a single, also known as addition, in primal numbers is not very different, except that the values cannot be greater than 9, because from there the primal number cycle is repeated and we would start again by 1, so when two or more primal numbers are added if the result is greater than 9, to this result will be applied the Status of Marrero, to obtain his true primal.

Next, the Mint Theorem will be presented for the primal sum

$$\forall\, Z_{\phi a} + Z_{\phi b} = Z_{\phi c} \cong v_a + v_b = v_c$$

Let's do some examples to demonstrate:

$$9_\phi + 2_\phi = 2_\phi \cong 18_9 + 11_2 = 29_2$$

This example is perfect for testing the neutrality of the Primal ϕ^9

$$\forall\, Z_\phi + 9_\phi = Z_\phi$$

This is reflected in the integers of the matrix because as observed in the example when adding a number of ϕ^9 with one of ϕ^2 the result will give us a number ϕ^9 always, in this case, it was 29.

Now we are going to test the associative property for the sum

$$Z_{\phi a}, Z_{\phi b}, Z_{\phi c} \in \{\zeta\} \rightarrow \left(Z_{\phi a} + Z_{\phi b}\right) + Z_{\phi c} = Z_{\phi a} + \left(Z_{\phi b} + Z_{\phi c}\right)$$

Examples:

A:$\left(4_\phi + 7_\phi\right) + 2_\phi = 4_\phi$

B:$4_\phi + \left(7_\phi + 2_\phi\right) = 4_\phi$

To show that there is a true congruence in the sum of integers, we will use different numbers of the same classes from both examples, which should give us two class numbers ϕ^4

A:$(22_4 + 16_7) + 11_2 = 49_4$

B:$13_4 + (97_7 + 272_2) = 382_4$

Before proceeding with the next operation, some cases must be taken into account when making the addition will be presented.

Case 1: $Z_{\phi a} + Z_{-\phi b} = Z_{\phi c}$

- When a positive number is added with a negative one, as a general rule, this becomes a subtraction and the sign of the greater number prevails, for a calculator this is logical, but there will be cases where the result in primal numbers is not reflected with what is described in a matrix.
An example would be

$$11_2 + (-8_8) = 3_3$$

Although everything seems normal, when performing a class transformation, we have that the operation on primal numbers would be as follows:

$$2_\phi + (-8_\phi) = 3_\phi$$

For a calculator, this would not make sense since it would give us a -6, let's proceed to solve this problem analytically.

We observe

$$2_\phi + (-8_\phi) = 3_\phi$$

We know that the result is positive

$$3_\phi$$

Therefore, we know that

$$rem(11{,}9) = 2_\phi > rem\,(-8{,}9) = -8_\phi$$

Proceed to use a new expression which would be "$\bar{\bar{Z}}_\phi$" the "$=$" indicate the number with the highest absolute value in the operation

$$\bar{\bar{2}}_\phi + (-8_\phi) = 3_\phi$$

This is an algebraic manipulation, used to preserve the sense of congruence:

To proceed to the calculation to the primal that the arrow carries, one of the digits that represent the identity would be added either 0 or 9, it is indifferent, we would have something like this

$$\bar{\bar{2}}0_\phi + (-8_\phi) = 3_\phi \; or \; \bar{\bar{2}}9_\phi + (-8_\phi) = 3_\phi$$

Then

$$\bar{\bar{2}}0_\phi + (-8_\phi) = (12)^M \equiv \bar{\bar{2}}9_\phi + (-8_\phi) = (21)^M = 3_\phi$$

Now the case is resolved and we can say that:

$$2_\phi + (-8_\phi) = 3_\phi \cong 11_2 + (-8_8) = 3_3$$

Case 2: $Z_{\phi a} + Z_{-\phi b} = Z_{-\phi c}$

- We have the case

$$2_\phi + (-8_\phi) = -6_\phi$$

Such that they are the integers

$$2_2 + (-8_{-8}) = -6_{-6}$$

This case is simple and corresponds to a more logical result since the negative number is greater than the positive, it is a direct calculation and does not need a mathematical device to arrive at the true result.

Table 3.

Addition table in its Primal form

	ϕ^1	ϕ^2	ϕ^3	ϕ^4	ϕ^5	ϕ^6	ϕ^7	ϕ^8	ϕ^9
ϕ^1	2_ϕ	3_ϕ	4_ϕ	5_ϕ	6_ϕ	7_ϕ	8_ϕ	9_ϕ	1_ϕ
ϕ^2	3_ϕ	4_ϕ	5_ϕ	6_ϕ	7_ϕ	8_ϕ	9_ϕ	1_ϕ	2_ϕ
ϕ^3	4_ϕ	5_ϕ	6_ϕ	7_ϕ	8_ϕ	9_ϕ	1_ϕ	2_ϕ	3_ϕ
ϕ^4	5_ϕ	6_ϕ	7_ϕ	8_ϕ	9_ϕ	1_ϕ	2_ϕ	3_ϕ	4_ϕ
ϕ^5	6_ϕ	7_ϕ	8_ϕ	9_ϕ	1_ϕ	2_ϕ	3_ϕ	4_ϕ	5_ϕ
ϕ^6	7_ϕ	8_ϕ	9_ϕ	1_ϕ	2_ϕ	3_ϕ	4_ϕ	5_ϕ	6_ϕ
ϕ^7	8_ϕ	9_ϕ	1_ϕ	2_ϕ	3_ϕ	4_ϕ	5_ϕ	6_ϕ	7_ϕ
ϕ^8	9_ϕ	1_ϕ	2_ϕ	3_ϕ	4_ϕ	5_ϕ	6_ϕ	7_ϕ	8_ϕ
ϕ^9	1_ϕ	2_ϕ	3_ϕ	4_ϕ	5_ϕ	6_ϕ	7_ϕ	8_ϕ	9_ϕ

In this table, the cycle of infinite repetitions of the primal values can be clearly appreciated.

6.2.2. Subtract Primal

Subtraction is defined as the operation that subtracts values from a number to obtain a result.

$$Z_{\phi a} - Z_{\phi b} = Z_{\phi c}$$

Acuña's theorem will obtain the following form

$$\forall\, Z_{\phi a} - Z_{\phi b} = Z_{\phi c} \cong v_a - v_b = v_c$$

Let's exemplify

$$8_\phi - 2_\phi = 6_\phi \cong 17_8 - 11_2 = 6_6$$

A then we study some cases that can occur when the operation is carried

Case 1: $Z_{\phi a} - Z_{\phi b} = Z_{\phi c} \neq Z_{\phi a} - Z_{\phi b} = Z_{-\phi d}$

- As we know the subtraction is not commutative, the order of the factors if it alters the product, then the results of primal numbers will be different

Example:

A:$2_\phi - 7_\phi = 4_\phi \cong 200_2 - 25_7 = 175_4$

As can be seen, the primal number of the lowest class represents an integer with an absolute value greater than the highest class number.
It is different if the integer values were different

B:$2_\phi - 7_\phi = -5_\phi \cong 20_2 - 25_7 = -5_{-5}$

Which means that the size of the integers that are going to be represented in primal numbers must always be taken into account, we can use the "Zi_ϕ" as in the case we studied in the sum to mark which is the number with the highest absolute value and make problem resolution easier.

Case 2: $Z_{\phi a} - Z_{\phi a} = Z_{\pm 9} \neq Z_{\phi a} - Z_{\phi a} = 0$

- As shown in the previous example, the value of the numbers always affects the transformation to primal numbers at the time of operation and in the case of subtracting two primal numbers that have the same value there is no difference, but we will always obtain three results ϕ^9, ϕ^{-9} or 0, these results are conditional.

Example:

$$4_\phi - 4_\phi = 9_\phi$$

This case occurs when two numbers that are subtracted the difference in size is not so much to produce a negative number.

$$4_\phi - 4_\phi = 9_\phi \cong 40_4 - 22_4 = 18_4$$

The following example would be

$$4_\phi - 4_\phi = -9_\phi$$

This case occurs when the difference is so great that the value is negative

$$4_{\phi} - 4_{\phi} = -9_{\phi} \cong 4_4 - 40_4 = -36_{-9}$$

The third and last case is

$$4_{\phi} - 4_{\phi} = \emptyset$$

This only happens when yes, and only if, the integer values being represented are the same.

$$4_{\phi} - 4_{\phi} = \emptyset \equiv 94_4 - 94_4 = 0$$

This case is represented in this form:

$$\forall\, Z_{\phi a} - Z_{\phi b} = 0 \; if \; and \; only \; if \;\; Z_{\phi a} = \; Z_{\phi b} \mid v_a = v_b$$

To the representation on a table we going to ignore this case, because we are only interested in results with integer values.
On the table you will see an interception between positive and negative numbers due to the interconnection between them.

This is related to the example given in the addition part (cap 6.2.1.) in Case 1, where the amount of the integer determines not only its sign, it also determines its class.

Examples using the class ϕ^9

We will use $\bar{\bar{\bar{Z}}}_{\phi}$ notation, as explained above to denote which number has the highest absolute value in the operation.

We get the following results table 4.

$\phi^9 - \phi^k \mid \lvert v_9 < v_{\phi} \rvert$	$\phi^9 - \phi^k \mid \lvert v_9 > v_{\phi} \rvert$
$9_{\phi} - \bar{\bar{\bar{1}}}_{\phi} = -1_{\phi}$	$\bar{\bar{\bar{9}}}_{\phi} - 1_{\phi} = 8_{\phi}$
$9_{\phi} - \bar{\bar{\bar{2}}}_{\phi} = -2_{\phi}$	$\bar{\bar{\bar{9}}}_{\phi} - 2_{\phi} = 7_{\phi}$
$9_{\phi} - \bar{\bar{\bar{3}}}_{\phi} = -3_{\phi}$	$\bar{\bar{\bar{9}}}_{\phi} - 3_{\phi} = 6_{\phi}$
$9_{\phi} - \bar{\bar{\bar{4}}}_{\phi} = -4_{\phi}$	$\bar{\bar{\bar{9}}}_{\phi} - 4_{\phi} = 5_{\phi}$
$9_{\phi} - \bar{\bar{\bar{5}}}_{\phi} = -5_{\phi}$	$\bar{\bar{\bar{9}}}_{\phi} - 5_{\phi} = 4_{\phi}$
$9_{\phi} - \bar{\bar{\bar{6}}}_{\phi} = -6_{\phi}$	$\bar{\bar{\bar{9}}}_{\phi} - 6_{\phi} = 3_{\phi}$
$9_{\phi} - \bar{\bar{\bar{7}}}_{\phi} = -7_{\phi}$	$\bar{\bar{\bar{9}}}_{\phi} - 7_{\phi} = 2_{\phi}$
$9_{\phi} - \bar{\bar{\bar{8}}}_{\phi} = -8_{\phi}$	$\bar{\bar{\bar{9}}}_{\phi} - 8_{\phi} = 1_{\phi}$
$9_{\phi} - \bar{\bar{\bar{9}}}_{\phi} = -9_{\phi}$	$\bar{\bar{\bar{9}}}_{\phi} - 9_{\phi} = 9_{\phi}$

Let's make an example of the first row:

$$90_9 - \overline{\overline{\overline{100}}}_1 = -10_{-1} \neq \overline{\overline{\overline{90}}}_9 - 10_1 = 80_8$$

In subtraction the same operation has two results unless you know which number has the highest absolute value, in the table we will place this condition and differentiate the results with the gray color for the negatives.
And we going to omit all the cases that are $Z_{\phi a} - Z_{\phi b} = 0$
Table 5.

Subtraction table in its Primal form

	ϕ^1		ϕ^2		ϕ^3		ϕ^4		ϕ^5		ϕ^6		ϕ^7		ϕ^8		ϕ^9	
ϕ^1	-9_ϕ	9_ϕ	-8	1_ϕ	-7_ϕ	2_ϕ	-6_ϕ	3_ϕ	-5_ϕ	4_ϕ	-4_ϕ	5_ϕ	-3_ϕ	6_ϕ	-2_ϕ	7_ϕ	-1_ϕ	8_ϕ
ϕ^2	-1_ϕ	8_ϕ	-9_ϕ	9_ϕ	-8_ϕ	1_ϕ	-7_ϕ	2_ϕ	-6_ϕ	3_ϕ	-5_ϕ	4_ϕ	-4_ϕ	5_ϕ	-3_ϕ	6_ϕ	-2_ϕ	7_ϕ
ϕ^3	-2_ϕ	7_ϕ	-1_ϕ	8_ϕ	-9_ϕ	9_ϕ	-8_ϕ	1_ϕ	-7_ϕ	2_ϕ	-6_ϕ	3_ϕ	-5_ϕ	4_ϕ	-4_ϕ	5_ϕ	-3_ϕ	6_ϕ
ϕ^4	-3_ϕ	6_ϕ	-2_ϕ	7_ϕ	-1_ϕ	8_ϕ	-9_ϕ	9_ϕ	-8_ϕ	1_ϕ	-7_ϕ	2_ϕ	-6_ϕ	3_ϕ	-5_ϕ	4_ϕ	-4_ϕ	5_ϕ
ϕ^5	-4_ϕ	5_ϕ	-3_ϕ	6_ϕ	-2_ϕ	7_ϕ	-1_ϕ	8_ϕ	-9_ϕ	9_ϕ	-8_ϕ	1_ϕ	-7_ϕ	2_ϕ	-6_ϕ	3_ϕ	-5_ϕ	4_ϕ
ϕ^6	-5_ϕ	4_ϕ	-4_ϕ	5_ϕ	-3_ϕ	6_ϕ	-2_ϕ	7_ϕ	-1_ϕ	8_ϕ	-9_ϕ	9_ϕ	-8_ϕ	1_ϕ	-7_ϕ	2_ϕ	-6_ϕ	3_ϕ
ϕ^7	-6_ϕ	3_ϕ	-5_ϕ	4_ϕ	-4_ϕ	5_ϕ	-3_ϕ	6_ϕ	-2_ϕ	7_ϕ	-1_ϕ	8_ϕ	-9_ϕ	9_ϕ	-8_ϕ	1_ϕ	-7_ϕ	2_ϕ
ϕ^8	-7_ϕ	2_ϕ	-6_ϕ	3_ϕ	-5_ϕ	4_ϕ	-4_ϕ	5_ϕ	-3_ϕ	6_ϕ	-2_ϕ	7_ϕ	-1_ϕ	8_ϕ	-9_ϕ	9_ϕ	-8_ϕ	1_ϕ
ϕ^9	-8_ϕ	1_ϕ	-7_ϕ	2_ϕ	-6_ϕ	3_ϕ	-5_ϕ	4_ϕ	-4_ϕ	5_ϕ	-3_ϕ	6_ϕ	-2_ϕ	7_ϕ	-1_ϕ	8_ϕ	-9_ϕ	9_ϕ

In this table, you can see how the previously explained cases work, where negative numbers are denoted in gray, and white when the positive is numerically greater and preserves the positive cycle.

6.2.3 Primal Multiplication

We define Primal multiplication as

$$Z_{\phi a} * Z_{\phi b} = Z_{\phi c}$$

Again we adapt the theorem to this form

$$\forall\, Z_{\phi a} * Z_{\phi b} = Z_{\phi c} \cong v_a * v_b = v_c$$

It would be redundant to exemplify each result individually since when the primal sum is fulfilled, the multiplication should not be an exception at all, so for practical terms, a multiplication table is shown that shows all the possible results and that we will take as a reference to exemplifying.

Table 6.

Multiplication table in its Primal form

	ϕ^1	ϕ^2	ϕ^3	ϕ^4	ϕ^5	ϕ^6	ϕ^7	ϕ^8	ϕ^9
ϕ^1	1_ϕ	2_ϕ	3_ϕ	4_ϕ	5_ϕ	6_ϕ	7_ϕ	8_ϕ	9_ϕ
ϕ^2	2_ϕ	4_ϕ	6_ϕ	8_ϕ	1_ϕ	3_ϕ	5_ϕ	7_ϕ	9_ϕ
ϕ^3	3_ϕ	6_ϕ	9_ϕ	3_ϕ	6_ϕ	9_ϕ	3_ϕ	6_ϕ	9_ϕ
ϕ^4	4_ϕ	8_ϕ	3_ϕ	7_ϕ	2_ϕ	6_ϕ	1_ϕ	5_ϕ	9_ϕ
ϕ^5	5_ϕ	1_ϕ	6_ϕ	2_ϕ	7_ϕ	3_ϕ	8_ϕ	4_ϕ	9_ϕ
ϕ^6	6_ϕ	3_ϕ	9_ϕ	6_ϕ	3_ϕ	9_ϕ	6_ϕ	3_ϕ	9_ϕ
ϕ^7	7_ϕ	5_ϕ	3_ϕ	1_ϕ	8_ϕ	6_ϕ	4_ϕ	2_ϕ	9_ϕ
ϕ^8	8_ϕ	7_ϕ	6_ϕ	5_ϕ	4_ϕ	3_ϕ	2_ϕ	1_ϕ	9_ϕ
ϕ^9	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ

Each of the columns represents tables to multiply the 1 to 9, except that instead of putting the results placed once numbers primal resulting from each operation, in column 9 observe that are all 9_ϕ this makes sense with the previously said that it Z_9 has behaviors similar to 0.

$$\forall Z_\phi * Z_9 = Z_9$$

If you wonder why it only has the results of nine rows, it is because from the tenth row for each column the pattern repeats itself.

To summarize, this table represents all the possible solutions of the multiplication that, when transformed into their primal form, acquire these values.

Examples:

A:$1_\phi * 4_\phi = 4_\phi \cong 19_1 * 31_4 = 589_4$

B:$7_\phi * 9_\phi = 9_\phi \cong 34_7 * 81_9 = 2754_9$

After observing the behavior of the numbers with the operations and the fact that we can use the primal numbers to replace the integers and in this way be able to study their behavior, we will proceed to the demonstration of the prediction that we are in the presence of a group.

6.2.4 Primal Power

If you can multiply and show a box then powers are possible, so we will proceed to build a class enhancement box.

Then we define that

For a $k \in \mathbb{Z} \,/\, if\ (\phi^k)^n \rightarrow (\phi^k)^n = \phi^{k^n}$

Table 7.

Table of powers in its Primal form.

	ϕ^{k^2}	ϕ^{k^3}	ϕ^{k^4}	ϕ^{k^5}	ϕ^{k^6}	ϕ^{k^7}	ϕ^{k^8}	ϕ^{k^9}	$\phi^{k^{10}}$	$\phi^{k^{11}}$	$\phi^{k^{12}}$	$\phi^{k^{13}}$	$\phi^{k^{14}}$	$\phi^{k^{15}}$
ϕ^1	1_ϕ	1_ϕ	1_ϕ	1_ϕ	1_ϕ	1_ϕ	1_ϕ	1_ϕ	1_ϕ	1_ϕ	1_ϕ	1_ϕ	1_ϕ	1_ϕ
ϕ^2	4_ϕ	8_ϕ	7_ϕ	5_ϕ	1_ϕ	2_ϕ	4_ϕ	8_ϕ	7_ϕ	5_ϕ	1_ϕ	2_ϕ	4_ϕ	8_ϕ
ϕ^3	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ
ϕ^4	7_ϕ	1_ϕ	4_ϕ	7_ϕ	1_ϕ	4_ϕ	7_ϕ	1_ϕ	4_ϕ	7_ϕ	1_ϕ	4_ϕ	7_ϕ	1_ϕ
ϕ^5	7_ϕ	8_ϕ	4_ϕ	2_ϕ	1_ϕ	5_ϕ	7_ϕ	8_ϕ	4_ϕ	2_ϕ	1_ϕ	5_ϕ	7_ϕ	8_ϕ
ϕ^6	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ
ϕ^7	4_ϕ	1_ϕ	7_ϕ	4_ϕ	1_ϕ	7_ϕ	4_ϕ	1_ϕ	7_ϕ	4_ϕ	1_ϕ	7_ϕ	4_ϕ	1_ϕ
ϕ^8	1_ϕ	8_ϕ	1_ϕ	8_ϕ	1_ϕ	8_ϕ	1_ϕ	8_ϕ	1_ϕ	8_ϕ	1_ϕ	8_ϕ	1_ϕ	8_ϕ
ϕ^9	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ	9_ϕ

In this table we can see all the results for the powers from the square to the power 15, a cycle of results can also be observed when the cycle is repeated the table becomes grayer, this phenomenon also occurs with the rest of the operations, the cycle of result classes is reflected in those primal numbers, which is a property that allows us to know what the primal numbers will be to raise a class to n-power and now we know that there are only six power models since the others will be repeated in an orderly and cyclical way to infinity.

Examples:

$(12_3)^3 = 1728_9 \cong (Z_3)^3 = Z_9$

$(45_9)^{11} = 1.532.278.301.220.703.125_9 \cong (Z_9)^{11} = Z_9$

$(2_2)^8 = 256_4 \cong (Z_2)^8 = Z_4$

6.3. Demonstration of the existence of a Group $\{\zeta_\phi \odot\}$

We define a group as a set no vacuum that an operation $\odot$ has the following properties:

- It is closed for operation

$$\phi^a \odot \phi^b = (\phi \odot \phi)^{(a \odot b)}$$

Let's suppose

$$(\phi \odot \phi)^{(a \odot b)} = (\phi' \odot \phi')^{(a' \odot b')}$$

Then, if

$$Z_{\phi a}, Z'_{\phi a} \in \phi^a \rightarrow Z_{\phi a} \equiv Z'_{\phi a}\ (mod\ 9)$$

$$Z_{\phi b}, Z'_{\phi b} \in \phi^b \rightarrow Z_{\phi b} \equiv Z'_{\phi b}\ (mod\ 9)$$

Therefore

$$9|Z_{\phi a} - Z'_{\phi a} \rightarrow Z_{\phi a} - Z'_{\phi a} = 9q \;/\; q \in \mathbb{Z}$$

$$9|Z_{\phi b} - Z'_{\phi b} \rightarrow Z_{\phi b} - Z'_{\phi b} = 9s \;/\; s \in \mathbb{Z}$$

Furthermore, by definition, we have to

$$(\phi \circledcirc \phi)^{(a \circledcirc b)} = \{v \in \mathbb{Z} \mid (Z_{\phi a} \circledcirc Z_{\phi B}) \ (mod\ 9)\}$$

Then we verify that

$$9|v - (Z_{\phi a} \circledcirc Z_{\phi B})$$

$$v - (Z_{\phi a} \circledcirc Z_{\phi B}) = 9t \;/\; t \in \mathbb{Z}$$

Then

$$\lambda)\ v = 9t + (Z_{\phi a} \circledcirc Z_{\phi B})$$

Starting from λ we will continue the proof for the addition operation and later we will continue from it for multiplication.

$$v = 9t + (Z_{\phi a} + Z_{\phi B})$$

$$v = 9t + (9q + Z'_{\phi a} + 9s + Z'_{\phi b})$$

$$v = 9(t + q + s) + (Z'_{\phi a} + Z'_{\phi b})$$

How $(t + q + s) \in \mathbb{Z}$, we finally get that

$$v = 9(t + q + s) + (Z'_{\phi a} + Z'_{\phi b}) \rightarrow v \equiv (Z'_{\phi a} + Z'_{\phi b})$$

Ergo

$$v \in (Z'_{\phi a} + Z'_{\phi b})$$

Now we continue from λ for multiplication

$$v = 9t + (Z_{\phi a} * Z_{\phi B})$$

$$v = 9t + \left[\left(9q + Z'_{\phi a}\right)\left(9s + Z'_{\phi b}\right)\right]$$

$$v = 9t + \left[\left(81qs + 9qZ'_{\phi b} + 9sZ'_{\phi a} + Z'_{\phi a} * Z'_{\phi b}\right)\right]$$

$$v = \left(9t + 81qs + 9qZ'_{\phi b} + 9sZ'_{\phi a}\right) + \left(Z'_{\phi a} * Z'_{\phi b}\right)$$

$$v = 9\left(t + 9qs + 9qZ'_{\phi b} + 9sZ'_{\phi a}\right) + \left(Z'_{\phi a} * Z'_{\phi b}\right)$$

How $\left(t + 9qs + 9qZ'_{\phi b} + 9sZ'_{\phi a}\right) \in \mathbb{Z}$, then

$$v = 9\left(t + 9qs + 9qZ'_{\phi b} + 9sZ'_{\phi a}\right) + \left(Z'_{\phi a} * Z'_{\phi b}\right) \to v \equiv \left(Z'_{\phi a} * Z'_{\phi b}\right)$$

Ergo

$$v \in \left(Z'_{\phi a} * Z'_{\phi b}\right)$$

- Associative property for sum in $\{\zeta_\phi\}$

Let

$$\phi^a, \phi^b, \phi^c \in \zeta_\phi \to [\phi^a + \phi^b] + \phi^c = \phi^a + [\phi^b + \phi^c]$$

6.4. Proofs:

Let's suppose

$$Z_{\phi a} \in \phi^a, Z_{\phi b} \in \phi^b, Z_{\phi c} \in \phi^c$$

Then

$$[\phi^a + \phi^b] + \phi^c = \left(Z_{\phi a} + Z_{\phi b}\right) + \phi^c$$
$$= \left[\left(Z_{\phi a} + Z_{\phi b} + Z_{\phi c}\right)^{a+b+c}\right]$$
$$= \left[\phi^a + \left(Z_{\phi b} + Z_{\phi c}\right)^{b+c}\right]$$
$$= \phi^a + \left[Z_{\phi b} + Z_{\phi c}\right]^{b+c}$$
$$= \phi^a[\phi^b + \phi^c]$$

- Property of existence of a Neutral Element (ϕ^9):

$$\phi^9 \in \{\zeta_\phi\} \; is\ the\ neutral\ element$$

6.5. Proofs:

$$\phi^a + \phi^9 = \{v \in \mathbb{Z} |\ v \equiv (Z_{\phi a} + 9)\ (mod\ 9)\}$$

Then

$$v - (Z_{\phi a} + 9) = 9t, for\ t \in\ \mathbb{Z}$$

$$v = 9t + (Z_{\phi a} + 9)$$

$$v = 9(t+1) + Z_{\phi a} \rightarrow v \equiv Z_{\phi a}\ (mod\ 9) \therefore v \in (\phi \odot \phi)^{a+9}$$

7. Theory group $\{\zeta_\phi +\}$

Prop: Existence of the Opposite Element.

$$\phi^{-a} \in \zeta_\phi \; is\ the\ opposite\ element.$$

7.1. Proof:

Let

$$(-Z_{\phi a}) \in \phi^{(-a)}$$

Then by definition, we can affirm that:

$$\phi^a + \phi^{(-a)} = \{v \in \mathbb{Z} | v \equiv [Z_{\phi a} + (-Z_{\phi a})\ (mod\ 9)]\}$$

$$v = 9t + [Z_{\phi a} + (-Z_{\phi a})], for\ t \in \mathbb{Z}$$

$$v = 9t + 0 \rightarrow v \equiv 0\ (mod\ 9) \rightarrow v \equiv 9\ (mod\ 9)$$

Ergo

$$v \in \phi^9$$

Finally

$$\phi^a + \phi^{(-a)} = \phi^9$$

7.2. Cancelative Law of the sum in ζ_ϕ

Let

$$\phi^a, \phi^b, \phi^c \in \zeta_\phi\ si\ \phi^a + \phi^c = \ \phi^b + \phi^c \rightarrow \phi^a = \phi^b$$

7.3. Proof:

As we well know, by definition:

$$(\phi+\phi)^{a+c}=(\phi+\phi)^{b+c}=\{v\in\mathbb{Z}|v\equiv Z_{\phi a}+Z_{\phi c}=Z_{\phi b}+Z_{\phi c}\ (mod\ 9)\}$$

From where

$$v-[(Z_{\phi a}+Z_{\phi c})=(Z_{\phi b}+Z_{\phi c})]=9t\quad for\ a\ t\in\mathbb{Z}$$

$$v=9t+[(Z_{\phi a}+Z_{\phi c})=(Z_{\phi b}+Z_{\phi c})]$$

$$v=9t+[(Z_{\phi a}+Z_{\phi c})+(-Z_{\phi c})=(Z_{\phi b}+Z_{\phi c})+(-Z_{\phi c})]$$

$$v=9t+[(Z_{\phi a}+Z_{\phi c}-Z_{\phi c})=(Z_{\phi b}+Z_{\phi c}-Z_{\phi c})]$$

$$v=9t+[Z_{\phi a}=Z_{\phi b}]\to v\equiv Z_{\phi a}=Z_{\phi b}\ (mod\ 9)\therefore v\in\phi^{a=b}$$

8. SúperSymmetry

Definition:

We proceed to define the Super Symmetry property that the matrix has, a property that shows the relationship that each element has within the different levels of the matrix theory.

$$v\equiv\phi^M_{(v)}\equiv\phi^k\equiv Z_\phi\ (mod\ 9)$$

8.1. Proof:

First, we will rewrite the definition of Super Symmetry in its following equivalent form:

$$v\equiv\phi^M_{(v)}\ (mod\ 9)\to\phi^M\equiv\phi^k\ (mod\ 9)\to\phi^k\equiv Z_\phi(mod\ 9)$$

So, we have that by definition of congruences

1) $v-\phi^M_{(v)}=9q,q\in\mathbb{Z}$
2) $\phi^M_{(v)}-\phi^k=9r,\ r\in\mathbb{Z}$
3) $\phi^k-Z_\phi=9s,s\in\mathbb{Z}$

Then, adding 1) and 2)

$$v-\phi^M_{(v)}+(\phi^M+\phi^k)=9q+9r$$

$$v-\phi^k=9(q+r)\to v\equiv\phi^k(mod\ 9)$$

Finally, adding this last resulting congruence with 3) we will obtain that

$$v-\phi^k+(\phi^k-Z_\phi)=9(q+r)+9s$$

$$v - Z_{\phi} = 9(q + r + s) \rightarrow v \equiv Z_{\phi}(mod\ 9), because\ (q, r, s) \in \mathbb{Z}$$

Therefore, by hypothetical syllogism.

$$v \equiv \phi_{(v)}^{M}\ (mod\ 9) \rightarrow \phi^{M} \equiv \phi^{k}\ (mod\ 9) \rightarrow \phi^{k} \equiv Z_{\phi}\ (mod\ 9)$$

9. Acuña's theorem proved

Previously mentioned, its idea is to function as a central theorem that denotes the behavior of the operations within the matrix, based on the already proven Primal group, and the property of Supersymmetry.
The objective of this theorem is to be able to support and summarize the why of all behavior of an integer with an operation that works in a matrix universe that allows us to reduce numbers towards a prominent element that we call the Primal Number and allow us to operate with them to be able to accurately predict which one. will be the class resulting from all these algebraic combinations.

9.1. Proof:
If

$$v_a, Z_{\phi a} \in \phi^{a} \rightarrow v_a \equiv Z_{\phi a}\ (mod\ 9)$$

$$v_b, Z_{\phi b} \in \phi^{b} \rightarrow v_b \equiv Z_{\phi b}\ (mod\ 9)$$

$$v_c, Z_{\phi c} \in \phi^{c} \rightarrow v_c \equiv Z_{\phi c}\ (mod\ 9)$$

Then by definition

$$(\phi \odot \phi)^{a+b=c} = \left\{x \in \mathbb{Z} |\ x \equiv Z_{\phi a} + Z_{\phi b} = Z_{\phi c}\ (mod\ 9)\right\}$$

From where

$$x - \left(Z_{\phi a} + Z_{\phi b} = Z_{\phi c}\right) = 9t, \qquad for\ t\ \in \mathbb{Z}$$

$$x = 9t + \left(Z_{\phi a} + Z_{\phi b} = Z_{\phi c}\right)$$

$$x = 9t + [(v_a - 9q) + (v_b - 9r) = (v_c - 9s)]\ \ for\ \ q, s, r, \in \mathbb{Z}$$

$$x = 9t + [(v_a + v_b) - 9(q + r) = v_c - 9s]$$

$$x = 9t + [(v_a + v_b) - 9(q + r) + 9s = v_c]$$

$$x = 9t + [9(-q - r + s) + (v_a + v_b) = v_c]$$

$$x = 9(-q - r + s + t) + (v_a + v_b) = v_c$$

$$\rightarrow x \equiv v_a + v_b = v_c\ \ (mod\ 9)$$

Ergo

$$x \in (\phi \odot \phi)^{a+b=c}$$

Conclusions

The study of the matrixE_{ϕ}^{9} was a complex work due to the direct clash with the theories of current mathematics, we can highlight its property of reducing numbers, operating them, and observing their behavior, also that it wanders between the theory of numbers and sets, that only studies the whole numbers and that is why we only see discrete results, or that for group theory poses a challenge, since it is a group with multiple neutrality thus breaking the pre-conception of group theory, in addition to a prediction it is possible that this can be applied for more complex operations, it can be given use to its ease of building analytical tables to study probabilities of obtaining certain kinds of certain operations.

Primal algebra describes with infinite loops of results how operations behave with discrete results, the application for this is limited by the imagination of whoever uses the tool.

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Eduardo J. Acuña. T. “*Superbia custodit nos stantes.”*

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