Universidad del Atlántico
MATUA
Revista MATUAPáginas: 1-18
Facultad de Ciencias Básicas
©Programa de Matemáticas
Vol. VII , № 1, (2020)

Received 16 Jun 2020; Approved 21 Dec 2020

GENERALIZATION OF THE CARTESIAN COORDINATES OF THE CYCLOGON CURVE FOR REGULAR POLYGONS OF n SIDES

GENERALIZACIÓN DE LAS COORDENADAS CARTESIANAS DE LA CURVA CICLOGÓN PARA POLÍGONOS REGULARES DE n LADOS

[1] **Jorge Jhonattan Castellanos Sosa**

Universidad de Huelva - Universidad Distrital Francisco José de Caldas
jorgejhonattan.castellanos@alu.uhu.es
ppocho.1214@gmail.com

Abstract

This work shows the procedure to obtain the generalization of the Cartesian coordinates of the Cyclogon with regular polygons of n sides. This is a curve that is obtained from the trace of a point belonging to the polygon when it is rotated on a horizontal plane, this geometric place is derived from the cycloid, so the angle Φ in charge of the rotation of the polygon is taken into account for its modeling, reaching to deductions like, that when this angle has given a cycle of 360 °, then the route in x of the curve is equal to the perimeter of the figure. For this application, tools such as the sinus theorem, the length of a curve, the length of an arc and the equation of the line are used, that is why they are immersed in authors such as Baldor (2004), Purcell, Varberg and Rigdon (2007), Lehmann (1989), and Sullivan (2006), among others.

Keywords: Cyclogon, Analytic Geometry, Cycloid, Dynamic Geometry.

Resumen

Este trabajo muestra el procedimiento para obtener la generalización de las coordenadas cartesianas del Ciclogón con polígonos regulares de n lados. Esta es una curva que se obtiene del rastro de un punto perteneciente a al polígono cuando este rota sobre un plano horizontal, este lugar geométrico deriva de la cicloide, así que se tiene en cuenta para su modelación el ángulo Φ encargado de la rotación del polígono, llegando a deducciones como, que cuando dicho ángulo haya dado un ciclo de 360°, entonces el recorrido en x de la curva es igual al perímetro de la figura. Para esta finalidad se utilizan herramientas como el teorema del seno, la expresión de la longitud de una curva, la longitud de un arco y la ecuación de la recta, es por ello que estuvieron inmersos autores como Baldor (2004), Purcell, Varberg y Rigdon (2007), Lehmann (1989), y Sullivan (2006), entre otros.

Palabras Clave: Ciclogón, Geometría Analítica, Cicloide, Geometría Dinámica.

1

E-mail address: jorgejhonattan.castellanos@alu.uhu.es, ppocho.1214@gmail.com

1. Justification

The studies carried out so far about the cyclogon are around the areas involved in the rotation of the polygons, this is how Apostol and Mnatsakanian (2012) indicate things like, that the area left by the cyclogon is three times the area of the regular polygon , or on the other hand, if you want to make a path through which a square or hexagonal wheel can pass, that is, a regular polygon, what would that path be like? but now the central point of the polygon is following a parallel line with respect to the horizontal axis, this situation is solved with catenary paths. However, a study has not been carried out that establishes the Cartesian equations of the cycloid, which is why this paper states these equations so necessary for subsequent analysis of the curve, which can also contribute to the modeling of other types of curves, in addition to its application in mechanics as the cycloid does.

2. Theoretical framework

Since the seventeenth century, the cycloid has been a curve widely studied by different individuals over time, mostly physicists and mathematicians. Many of those who worked on this brachistochronous and tautochron curve disputed its discovery, such as Nicolás de Cusa, Galileo Galilei, Marín Mersenne, among others (Fernández, 2019).

But, on the other hand, what happens when instead of a circle rotating on a horizontal plane were a polygon?; certainly the curve left by this is called a cyclogon, and the more sides the (regular) polygon has, the more the curve resembles being a cycloid. However, it can be done with both regular and irregular figures, it does not matter whether the point marked by the stroke is on the figure or not. Apostol and Mnatsakanian (2012) define this curve as: "when a polygonal disk rolls along a straight line, each vertex a curve we call cyclogon" (pg.67).

Several things have been obtained from the study of this curve, for example, for every cyclogon generated by a regular polygon we have to:

$$A = P + 2C$$

Where A denotes the area of the region above the line of the horizontal plane and below the cyclogonal curve, that is, the area that forms the cyclogon with respect to the horizontal axis, P is the area of the polygon, and C is the area of the circular section that circumscribes the polygon. Here the question is to know exactly what is the area left by the cyclogon, there Apóstol and Mnatsakanian (2012) show said demonstration on page 69 of their book "New Horizons in Geometry".

Also, Alsina and Nelsen (2010) show proofs about the area obtained by the rotation of the regular polygon, such as theorem 4.11:

"*When a regular polygon is rolled a line, the area of the polygonal cycloid generated by a vertex of the polygon is three times the area of the polygon*" (pg. 68)

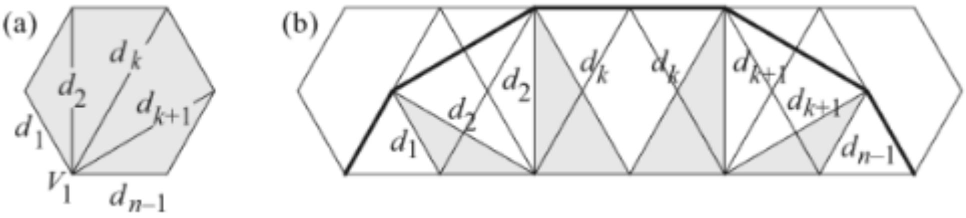

Illustration 1: Analysis used by Alsina and Nelsen (2010) to show that the area left by the cyclogon is three times the area of the polygon.

Another proof that these authors carry out is about the following theorem:

"When a regular polygon is rolled along a line, the length of the polygonal cycloid generated by a vertex of the polygon is four times the sum of the inradius and the circumradius of the polygon" (pg. 68)

For the most part, advanced geometry books discuss the areas involved with the cyclogon, but there is no Cartesian relationship with the emergence of the curve.

In addition, the curve that the polygon should have underneath (the shape of the plane) has also been studied so that when it turns on it, the center of the polygon travels a straight horizontal line parallel to the x axis. This appears in the museum of mathematics in New York with the square-wheeled bicycle.

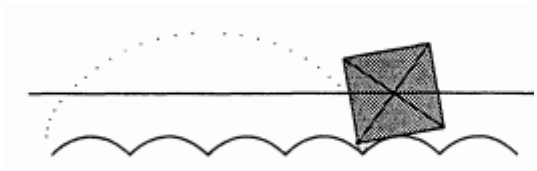

Illustration 2: Scheme that exemplifies the path of a square wheel through a plane full of sections of catenaries.

The curves of the road through which the bicycle must pass is determined by several sections of catenaries, which take into account the rotation of the wheel. Hall and Wagon (1992) showed that the expression of this curve for k-sided polygons was determined by:

$$\phi(x) = \frac{-2\,arctan\left(\frac{e^{-x}-k}{\sqrt{1-k^2}}\right)}{\sqrt{1-k^2}}$$

They indicated that the wheels of said "bicycle" cannot always be polygon-shaped, but rather geometric objects such as spirals, ellipses, cardioids, parabolas...

As mentioned above, beyond the analysis of the areas left by the cyclogon, a study has not been established that expresses this as a Cartesian function that relates the angle at which the figure rotates, with respect to the position of the point it generates the cyclogon. This is how the tools used for this were:

- The equation of the circumference:

Lehmann (1989) in his book "Analytical Geometry" mentions the ordinary equation of the circumference as:

$$(x-h)^2 + (y-k)^2 = r^2$$

Being the point G = (h, k) the coordinates of the center of the circumference, and r its radius (Pag 99).

- Sine theorem:

This is defined because the sides of a triangle are proportional to the sines of the opposite angles (Baldor, 2004). The expression that expresses this relationship is:

$$\frac{a}{sin\ (A)} = \frac{b}{sin\ (B)} = \frac{c}{sin\ (C)}$$

Being:

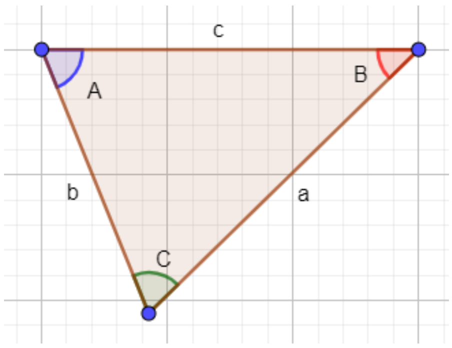

Illustration 3: In any type of triangle, this theorem can be applied.

- Arc of circumference with respect to its angle and radius:

Baldor (2004) announces that the measure of a circumference arc or circumference section is found by multiplying the radius of the circumference with the angle that determines the circumference section in radians, in this way the measure of the circumference arc is obtained:

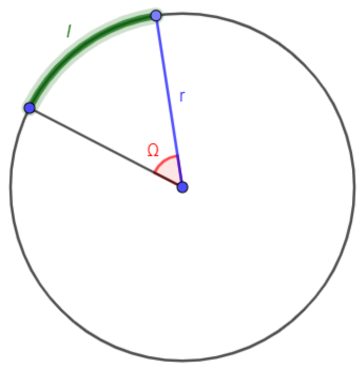

Illustration 4: The measure of the arc of circumference (l) is given by l = Ω r, with Ω in radians.

- Length of a curve:

Purcell, Varberg and Rigdon (2007), mention that:

Let f be continuously differentiable in $[a, b]$. For each x in (a, b), define $s(x)$like:

$$s(x) = \int_a^x \sqrt{1 + [f'(u)]^2} du$$

Then, s(x) gives the length of the arc of the curve f(u) from the point (a, f (a)) to (x, f (x)), then, by the fundamental theorem of calculus we have that :

$$s'(x) = \frac{ds}{dx} = \sqrt{1 + [f'(u)]^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

So, the arc length differential can be written as:

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

(pg. 298)

- Slope of the angle-generating segment in the unit circle

The slope of a line is the tangent of the angle of inclination. In these conditions $m = tan\,\theta$, being $\theta$the angle of inclination, and m the slope of the line. The slope of the line through two points $P(x_1, y_1)$, and $Q = (x_2, y_2)$, is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = tan\,\theta$$

(Kindle, 1969. Pg. 2)

- Equation of a line

Sullivan (2006) indicates that the slope of a line that passes through two different points $P(x_1, y_1)$, and $Q = (x_2, y_2)$, with $x_1 \neq x_2$,is defined as:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

If $x_1 = x_2$ it is a vertical line, and the slope would not be defined. (pg. 181)

In this way, the equation of a line with slope m, and b as intersection with the y-axis is (also called, slope-ordered form of a line):

$$y = mx + b$$

(pg. 187)

Given these tools, we proceed to the explanation of obtaining the Cartesian function for the cyclogon curve with regular polygons of n sides.

3. Generalization of the Cartesian coordinates of the Cyclogon

The cycloid arises in the same way that a large part of geometric curves arise, due to the movement of circles. This is how it is formed with the geometric place, or trace that leaves a single point on some part of a circumference, when it is moving on a horizontal plane.

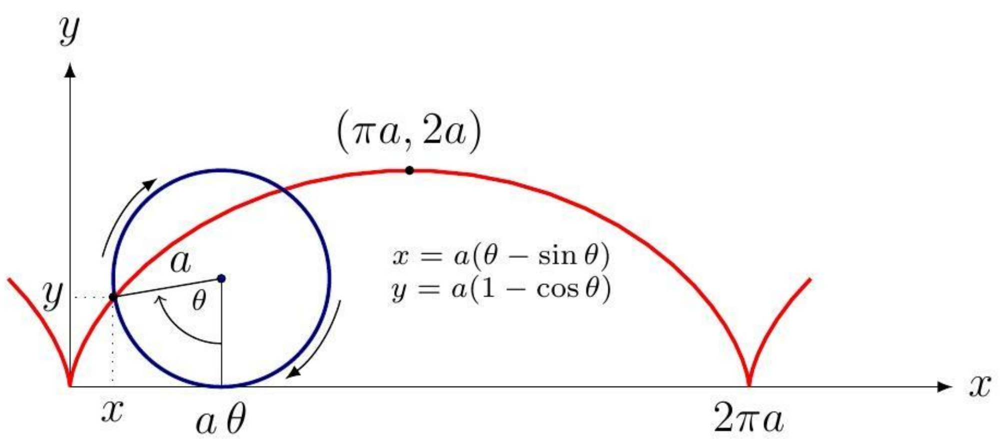

Illustration 5: Representation of obtaining a cycloid, together with its parametric equations.

Now, to know what a cyclogon is, we have to imagine that, instead of a circle rotating on the horizontal plane, it was a polygon with n number of sides.

The initial positions of the polygons will be "standing" by one of their vertices perpendicular to the horizontal axis, as follows:

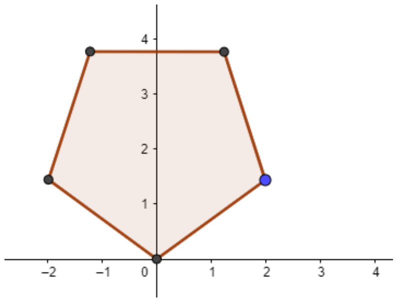

Illustration 6: Example of initial position of the polygon to obtain the curve.

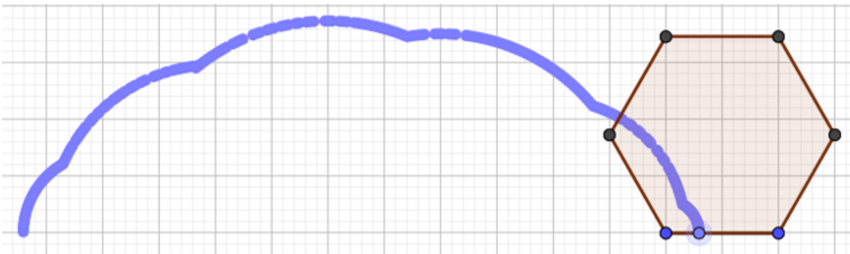

Illustration 7: Cyclogon generated by a regular hexagon.

Now, to begin modeling this situation, the center of the polygon must be taken into account, since, as in the cycloid, the center of the circumference is the one that helps parameterization and analysis due to the rotation it performs around its axis. But in this case, the center of the polygon does not travel in a straight line like the cycloid, parallel to the horizontal axis:

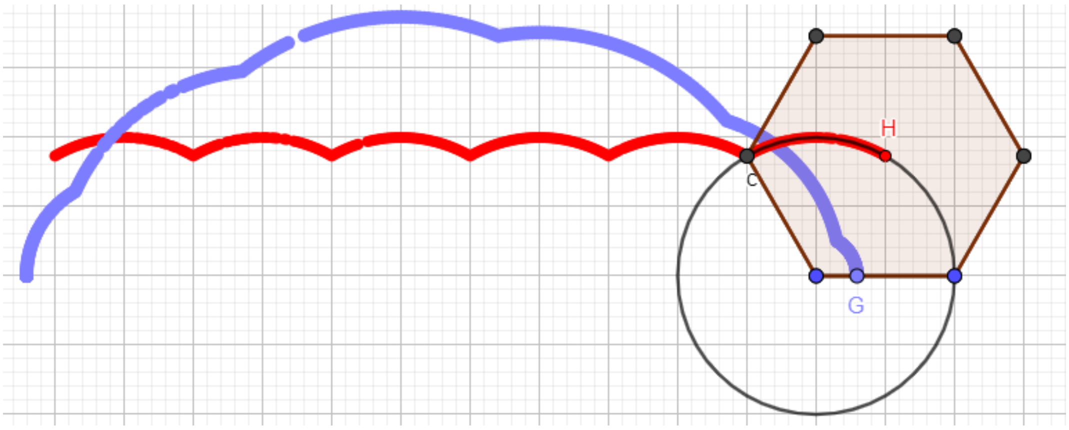

Illustration 8: Path left by the center of the polygon (red color) created by circumference sections.

The curved path in red is the one that leaves the center of the hexagon. They are clearly arcs of circumference whose radius is the distance that exists between the center of the polygon and one of its vertices (this will be the radius that will be taken into account later). In this way the question that arises is, how to model this curved path?

For this, the general equation of half a circle given its center (for positive quadrants in y) will be taken into account,

$$f(x) = \sqrt{(r^2 - (x-h)^2)} + k$$

Which follows from the ordinary equation of the circumference given by Lehmann (1989); where C = (h, k) which is the center of the half circle. On the other hand r will always be constant according to the regular polygon that is had and also the measure of its sides, in this way the expression that relates these elements thanks to Baldor (2004) is:

$$r = \frac{l \cdot sin\left(\frac{90(n-2)}{n}\right)}{sin\left(\frac{360}{n}\right)}$$

Being

l = The measure of one of the sides of the polygon

n = Number of sides of the polygon

The generalization of the curved path for a polygon with n sides would be as follows:

Being k = 0 because the centers are on the x axis.

$g_0(x) = \sqrt{(r^2 - (x-h)^2)}$, such that $-\frac{l}{2} < x \leq 0l + \frac{l}{2}$, $h = 0$

$g_1(x) = \sqrt{(r^2 - (x-h)^2)}$, such that $0l + \frac{l}{2} < x \leq 1l + \frac{l}{2}$, $h = l$

$g_2(x) = \sqrt{(r^2 - (x-h)^2)}$, such that $1l + \frac{l}{2} < x \leq 2l + \frac{l}{2}$, $h = 2l$

$g_3(x) = \sqrt{(r^2 - (x-h)^2)}$, such that $2l + \frac{l}{2} < x \le 3l + \frac{l}{2}$, $h = 3l$

$g_4(x) = \sqrt{(r^2 - (x-h)^2)}$, such that $3l + \frac{l}{2} < x \le 4l + \frac{l}{2}$, $h = 4l$

…

This is repeated n times, n being the number of sides of the polygon, since a cycle would be fulfilled, the expression would be as:

$$c(x) = \cup_{m=0}^{n} g_m(x) = \sqrt{(r^2 - (x-h)^2)}, \text{such that } (m-1)l + \frac{l}{2} < x \le ml + \frac{l}{2}, h = ml$$

With $m \in Z$

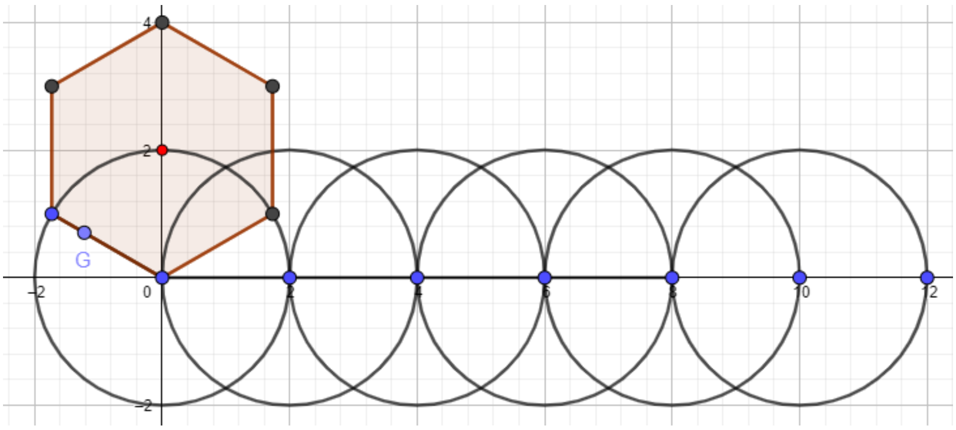

Illustration 9: The complete circles from which the path of the center of the polygon (H) arises.

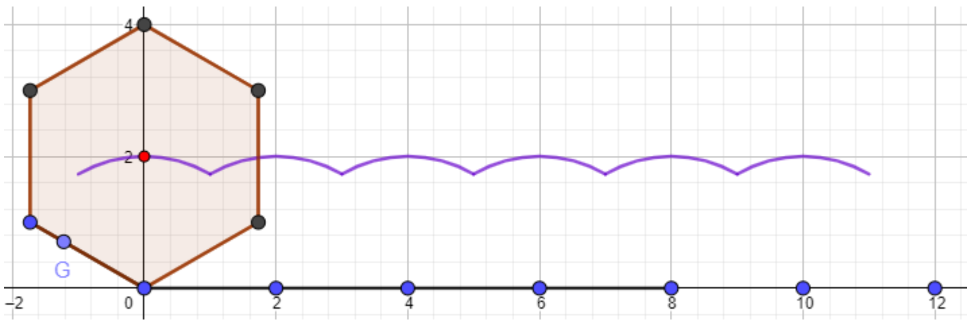

Illustration 10: Point curved path of the center of the polygon (H) in a complete 360 ° rotation (omitting the negative piece).

In this way, we have the path of the center of any regular polygon, therefore, we can give way to knowing how to find the exact position of the point which is going to leave the trace of the cyclogon (G) with respect to the angle of rotation of the polygon (ϕ). Therefore, it will be done in the following way:

As the initial position of the polygon is "standing", there is an angle ϕ that arises from the rotary movement of the polygon on the horizontal plane, with respect to this angle, the angles that are greater than this will be positive, and the smallest negative, is trivial, however, this will be taken into account to know the exact position of the point G belonging to the polygon.

Let's say that the point G that the cyclogon is going to leave is located at -40° from the initial position Φ= 0°(ω), so what would be its distance to the center (GH)? This would be a fixed position that is key in the system, despite the angle Φ when it elapse:

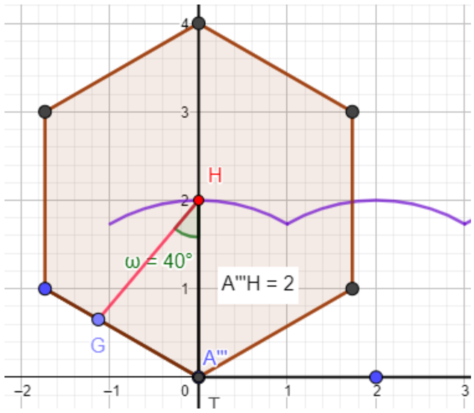

Illustration 11: Position of the generator point G of the cyclogon from an initial angle ω of -40 °

Generalizing for a polygon with n number of sides, the distance between G and the Center H (distance λ), would be given by the following analysis:

- If $|\omega| = m\left(\frac{360°}{n}\right)$, with $m \in N$, then the distance GH (λ) es equal to r.
- If $|\omega| \neq m\left(\frac{360°}{n}\right)$, with $m \in N$, then:

 If $0 < |\omega| < \left(\frac{360°}{n}\right)$,thanks to the sine theorem (Baldor, 2004) the equation is applied:

$$\lambda = GH = \frac{sin\left(\frac{90(n-2)}{n}\right) r}{sin\left(180 - \left(|\omega| + \left(\frac{90(n-2)}{n}\right)\right)\right)}$$

Where n = Number of sides of the polygon

It should be noted that, as the initial position of the polygon is symmetric, it does not matter if ω is positive or negative, the sign only indicates the direction where the point G is with respect to the angle Φ (A'''HT). Solving the equation for the given hexagon we have λ = GH≈1.76.

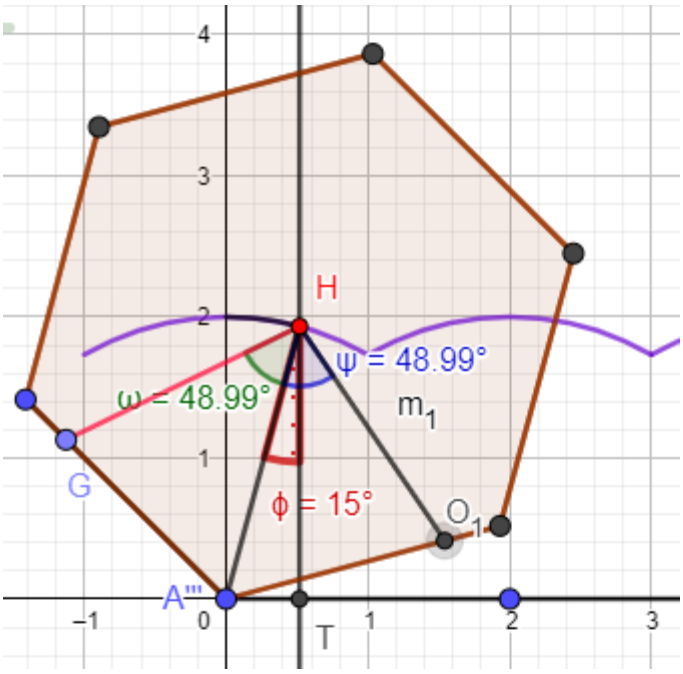

Illustration 12: The angle ω when it is negative (A'''HG), and when it is positive $A'''HO_1$ (ψ)

- But if $\left(\frac{360°}{n}\right) < |\omega|$, the following operation is performed:

$$|\omega| \div \frac{360}{n}$$

The result must be expressed as a mixed number of the form $q\frac{s}{p}$, with $q, n\ y\ p \in Z, y\ p \neq 0$
Then the proper fraction belonging to the mixed number is multiplied $\frac{s}{p}$ by $\frac{360}{n}$:

$$\frac{s}{p} \cdot \frac{360}{n} = \delta$$

Then simply apply:

$$\lambda = GH = \frac{sin\left(\frac{90(n-2)}{n}\right) r}{sin\left(180 - \left(\delta + \left(\frac{90(n-2)}{n}\right)\right)\right)}$$

In this way you can find any distance between point G and the center of the polygon (H).

Now the relationship between the angle (ϕ), which is the independent variable of the system, must be established with the x axis with respect to the centers of the polygon in order to know that, when the angle has a certain value, what would be the position of the center of the polygon and vice versa; This is how it is taken into account that the angles belonging to the circumference arcs of the center function c(x) also indicate the rotation of the figure, that is, if the figure has rotated 100 degrees (in the case of the hexagon), then the center of the polygon will be located in the third "mountain", because, since the first, as it is half, would be 30°, the second has 60°, and the other also has 60°, that is, the center would be located there:

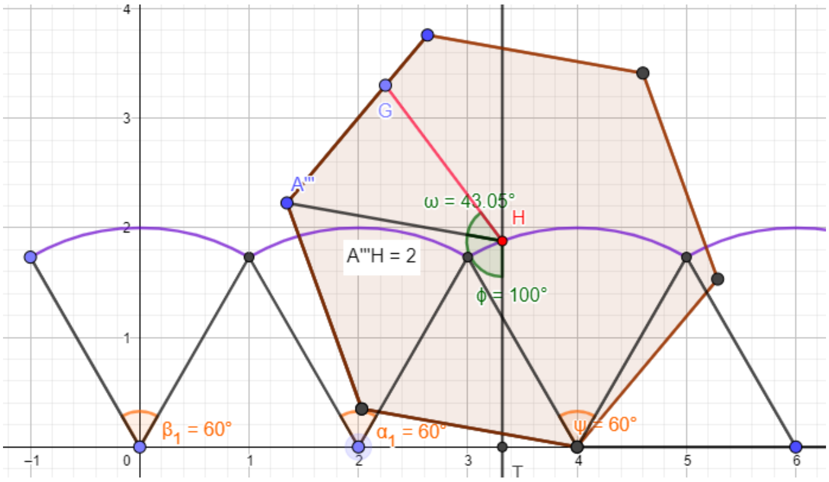

Illustration 13: When the angle Φ has 100 °, it will be located in the third "mountain" of the function c(x), each of those "ice cream cones" has an angle of $\frac{360°}{n}$.

In this way, each of the "mountains" represents the angle $360°$, so a relationship between the distance of each arc with respect to the angle can be implemented, and thus the position of the center would be involved.

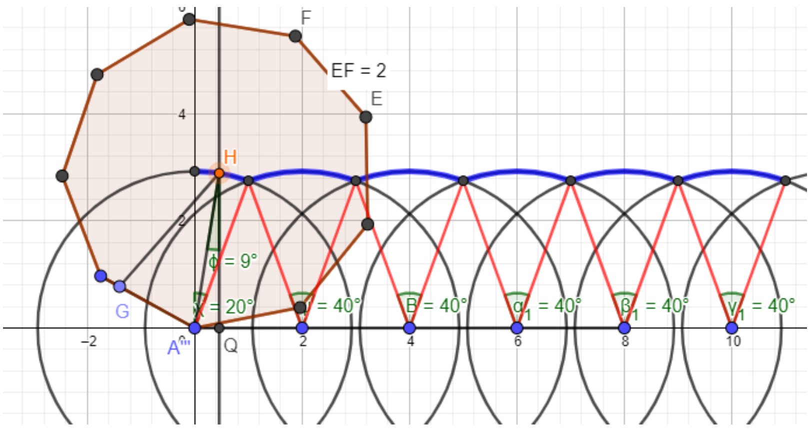

Illustration 14: Angles of the "ice cream cones" in a regular nonagon.

The equation that involves the measure of an arc with respect to the multiplication between the angle and the radius of the circular section will be applied (Baldor, 2004), together with the curve length formula (Purcell, Varberg and Rigdon, 2007) is obtained:

- The angle Φ must be in radians.
- Being also $c'(x) = \frac{h-x}{\sqrt{r^2-(h-x)^2}}$

$$\Phi \cdot r = \int_0^x \sqrt{1 + (c'(x))^2}dx$$

Then, clearing Φ, we have:

$$\Phi = \frac{\int_0^x \sqrt{1+(c'(x))^2}dx}{r}$$

Therefore, when x is cleared from the equation, we find the value of x belonging to the coordinate of the center of the polygon (H) with respect to the angle Φ. Next, the respective equations are shown for each section of motion of the polygon , or for each "mountain":

$$\Phi \cdot r = \int_0^x \sqrt{1+(c'(x))^2}dx, \text{ con: } 0 < \Phi \le 0\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2}$$

$$\Phi \cdot r = \int_{\frac{l}{2}}^x \sqrt{1+(c'(x))^2}dx, \text{ con: } 0\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2} < \Phi \le 1\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2}$$

$$\Phi \cdot r = \int_{l+\frac{l}{2}}^x \sqrt{1+(c'(x))^2}dx, \text{ con: } 1\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2} < \Phi \le 2\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2}$$

$$\Phi \cdot r = \int_{2l+\frac{l}{2}}^x \sqrt{1+(c'(x))^2}dx, \text{ con: } 2\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2} < \Phi \le 3\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2}$$

$$\Phi \cdot r = \int_{3l+\frac{l}{2}}^x \sqrt{1+(c'(x))^2}dx, \text{ con: } 3\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2} < \Phi \le 4\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2}$$

...

So, a respective correspondence is made between the function of the center *c(x)*, with the relationship established between the principal angle Φ and the x coordinate of H (*(x) H*):

$c(x)$	Relation between Φ and $(x)H$
$g_0(x) = \sqrt{(r^2-(x-h)^2)}$, such that: $-\frac{l}{2} < x \le 0l + \frac{l}{2}$, $h = 0$	$\Phi \cdot r = \int_0^x \sqrt{1+(c'(x))^2}dx$, with: $0 < \Phi \le 0\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2}$
$g_1(x) = \sqrt{(r^2-(x-h)^2)}$, such that: $0l + \frac{l}{2} < x \le 1l + \frac{l}{2}$, $h = l$	$\Phi \cdot r = \int_{\frac{l}{2}}^x \sqrt{1+(c'(x))^2}dx$, with: $0\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2} < \Phi \le 1\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2}$
$g_2(x) = \sqrt{(r^2-(x-h)^2)}$, such that: $1l + \frac{l}{2} < x \le 2l + \frac{l}{2}$, $h = 2l$	$\Phi \cdot r = \int_{l+\frac{l}{2}}^x \sqrt{1+(c'(x))^2}dx$, with: $1\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2} < \Phi \le 2\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2}$
$g_3(x) = \sqrt{(r^2-(x-h)^2)}$, such that: $2l + \frac{l}{2} < x \le 3l + \frac{l}{2}$, $h = 3l$	$\Phi \cdot r = \int_{2l+\frac{l}{2}}^x \sqrt{1+(c'(x))^2}dx$, with: $2\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2} < \Phi \le 3\left(\frac{90(n-2)}{n}\right) + \frac{\left(\frac{90(n-2)}{n}\right)}{2}$

$g_4(x) = \sqrt{(r^2-(x-h)^2)}$, such that: $3l+\frac{l}{2}< x \leq 4l+\frac{l}{2}, h=4l$	$\Phi \cdot r = \int_{3l+\frac{l}{2}}^{x} \sqrt{1+(c'(x))^2}dx$, with: $3\left(\frac{90(n-2)}{n}\right)+\frac{\left(\frac{90(n-2)}{n}\right)}{2}< \Phi \leq 4\left(\frac{90(n-2)}{n}\right)+\frac{\left(\frac{90(n-2)}{n}\right)}{2}$

Generalizing with m∈Z, where n is the number of sides of the regular polygon, and l is the measure of the side of the polygon:

$c(x) = \bigcup_{m=0}^{n} g_m(x) = \sqrt{(r^2-(x-h)^2)}$, $such\ that\ (m-1)l+\frac{l}{2}< x \leq ml+\frac{l}{2}, h=ml$	$\Phi \cdot r = \int_{(m-1)+\frac{l}{2}}^{x} \sqrt{1+(c'(x))^2}dx$, with: $(m-1)\left(\frac{90(n-2)}{n}\right)+\frac{\left(\frac{90(n-2)}{n}\right)}{2}< \Phi \leq m\left(\frac{90(n-2)}{n}\right)+\frac{\left(\frac{90(n-2)}{n}\right)}{2}$

In this way, given the independent variable Φ, we can get to know the exact position of point H, that is, the center of the regular polygon.

Now, the last thing to know is the exact coordinates of point G. Therefore, it is convenient to know what is the slope of the line that marks the angle Φ (A'''H), to later know the slope of the line GH that marks the angle ω. In this way, to know the slope of the line A’’’H is easy since it would be of the form:

$m_1 = tan\ (90-\Phi)$, with Φ in degrees (Kindle, 1969)

In this way the slope of GH is:

$m_2 = tan\ ((90-\Phi)+\omega)$, with Φ in degrees; although there is a small conditional of the sign to the left of the angle ω. This is positive if it goes counterclockwise with respect to line A’’’H, and is negative otherwise, something that had been mentioned in Illustration 11.

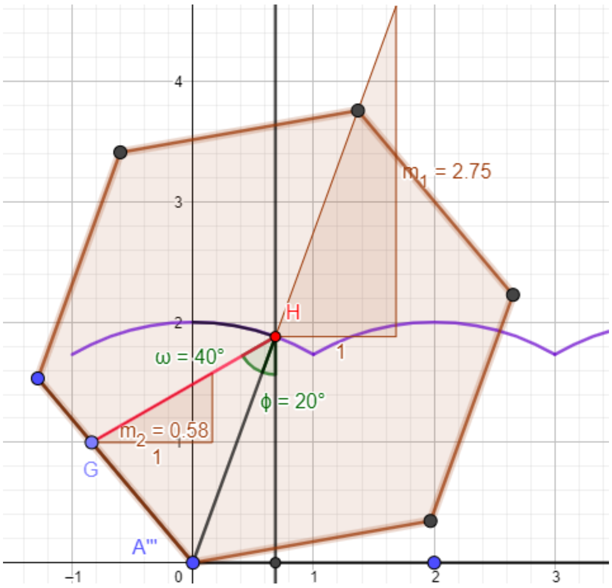

Illustration 15: Slopes of segments A'''H and GH, with respect to angle Φ and ω.

In this way, the function of the line that passes through G and H (Sullivan, 2006) is:

$$s(x) = m_2 x + (c(x) - m_2 x)$$
$$s(x) = tan\left((90 - \Phi) \pm \omega\right) x + \left(c(x) - \left(tan\left((90 - \Phi) \pm \omega\right)(x)H\right)\right)$$

 $(x(H))$ is the x coordinate of H as indicated above; remember that it can be found thanks to the angle.

On the other hand, since the exact distance between HG is known, the general equation of the half circumference (Lehmann, 1989), which has as center H and radius HG, is equated with the equation of the line that passes through G and H, This is how we will know the coordinates of point G:

$$\sqrt{\lambda^2 - ((x)H - x)^2} + c(x) = tan\left((90 - \Phi) \pm \omega\right) x + \left(c(x) - \left(tan\left((90 - \Phi) \pm \omega\right)(x)H\right)\right)$$
$$x = (x)H - \lambda \cos\left((90 - \Phi) \pm \omega\right) = (x)G$$

This is how we obtain the x-coordinate of point G. To find the y-coordinate $((y)G)$ simply replace the value of x in $s(x)$.

In this way, the coordinates of the point that generates the locus of the cyclogon for a regular polygon with n number of sides is:

$$G = \left(\left((x)H - \lambda \cos\left((90 - \Phi) \pm \omega\right)\right), \left(s(x)\right)\right)$$

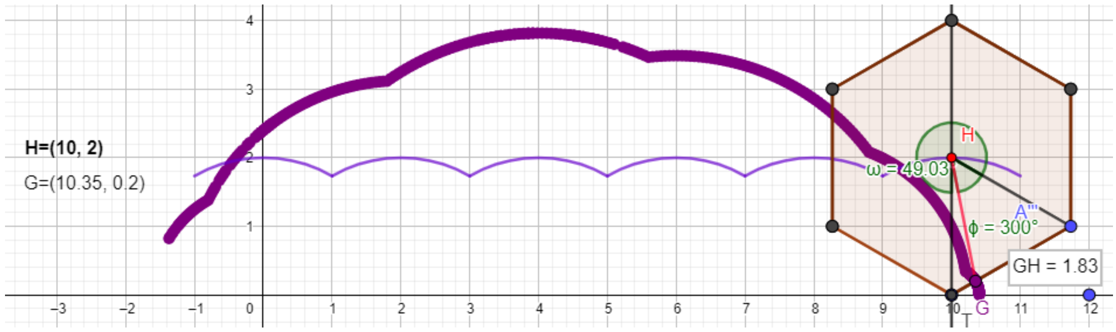

Illustration 16: Cyclogon generated by a regular hexagon with l = 2, the coordinates of G, H, angles ϕ and ω, and the measure of λ (GH).

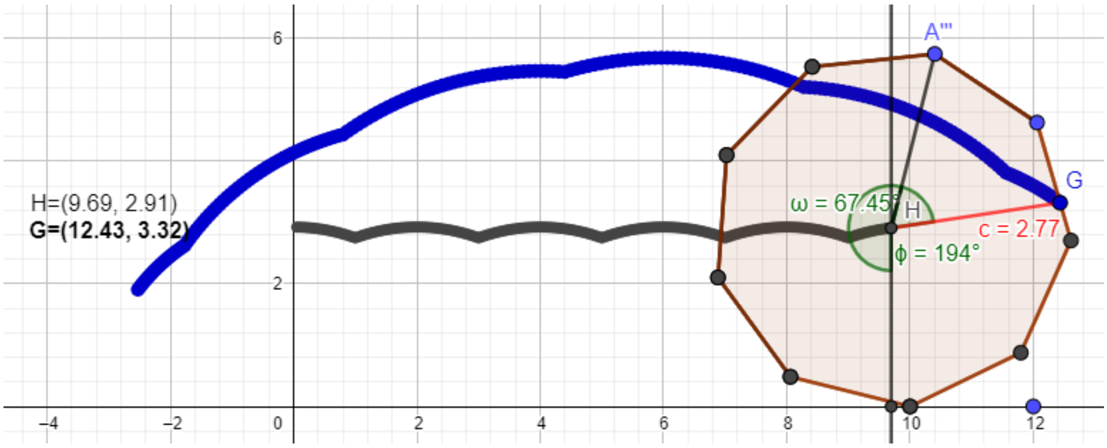

Illustration 17: Cyclogon generated by a regular nonagon with l = 2, indicating the coordinates of G, H, the angles ϕ and ω, and the measure of λ (GH).

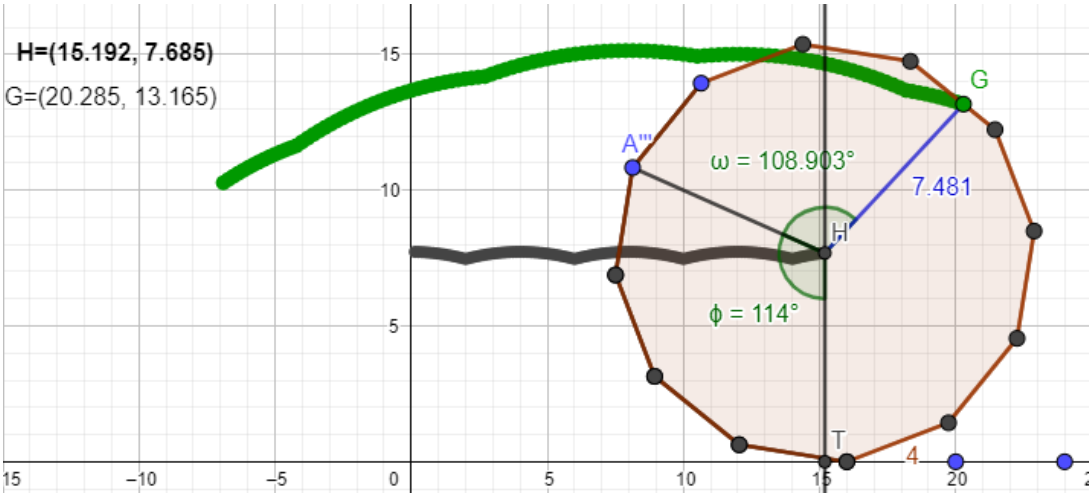

Illustration 18: Cyclogon generated by a regular dodecagon, indicating the coordinates of G, H, angles ϕ and ω, and the measure of λ (GH).

4. Conclusions

Obtaining the Cartesian expression of point G was certainly not an easy task, since several theorems and elements were involved that helped to cement the reasoning, however this result brings with it a series of conclusions derived from the process, among them it is obtained that :

Each time the regular polygon has more sides, the cyclone will look more and more like a cycloid. It is very similar to what happens in the exhaustion method carried out by Archimedes, since each time you have a regular polygon with a greater number of sides, it will look more like a circumference, this is how the cyclogon will tend to be each time plus a cycloid as the sides of the polygon increase.

One essential thing is that, if you want the G point to be outside or inside the polygon, you only have to increase or decrease the GH measure, call this measure Kiu. In this way, more complex cyclogons will be generated, but clearly, the more sides the polygon has, these trajectories will look like those left by a circular figure, with a G position greater or less than its radius.

It is important to mention that the total distance of a rotation that any regular polygon with n sides travels on the x axis when the cyclogon is generated is equal to the measure of its perimeter, in addition to being related to the 360 ° belonging to the rotation around the angle until it gets there.

Another thing is that the angle that determines the G point could be only negative, or if you want only positive, since it could cover the entire polygon in a complete rotation, that way the G point could be anywhere in the polygon .

The study of these curves contributes to a deeper understanding of their properties, which is easier if it is given in the Cartesian way because it is closer to the current forms of mathematical modeling and analysis, which is Analytical and Dynamic Geometry, for This establishes the Cartesian equations for later analysis in different fields. Also because their knowledge contributes to mechanics, in the same applications of the cycloid, but in more complex situations, together with physics, which acquires an important role for a subsequent analysis of its properties in the palpable world.

It can be noted that the infinite repetitions, or the cycles have a lot to do with what is found in the modeling of the cycloid, it reminds us of those fractal forms which are founded on certain structures of the universe, so it is convenient to continue with the modeling the world, although clearly we will never know everything about it.

Agradecimientos

Agradezco a Dios y a mí mismo por lograr avanzar en la construcción del conocimiento, y la utilidad de la vida. Además de mi profesor Edwin Carranza por el reto.

Referencia

Bibliography

[1] Alsina, C. Nelsen, R. (2010). Charming proofs a journey into elegant mathematics. Editorial The mathematical association of America (incorporated). United states.

[2] Apostol, T. Mnatsakanian, M. (2012). New Horizons in geometry. Mathematical association of America. Dolciani mathematical expositions # 47. South Korea.

[3] Baldor, A. (2004). Geometría plana y del espacio con una introducción a la trigonometría. Editorial Cultural S.A. México D.F.

[4] Fernández, I. (2019). Curvas planas con propiedades físicas o geométricas especiales y útiles. Tesis, Universidad de Cantabria, facultad de ciencias, España.

[5] Kindle, J. (1969). Teoría y problemas de Geometría analítica, plana y del espacio. Mc Graw-Hill. México D.F.

[6] Lehmann, C. (1989). Geometría Analítica. Editorial Limusa. México DF

[7] Leon Hall, L. Wagon, S. (1992) “Roads and Wheels.” Mathematics Magazine, Vol. 65, No. 5 pp. 283-301.

[8] Purcell, E. Varberg, D. Rigdon, S. (2007). Cálculo diferencial e integral. Editorial Pearson. México D.F.

[9] Sullivan, M. (2006). Álgebra y trigonometría, séptima edición. Pearson education. México D.F.