

SIMULACIÓN DESARROLLADA EN SIMULINK DE UN SISTEMA MECANICO
MASA-RESORTE-AMORTIGUADOR CON FUERZA EXTERNA VARIABLE

SIMULINK DEVELOPED SIMULATION OF A MASS-SPRING-SHOCK MECHANICAL
SYSTEM WITH VARIABLE EXTERNAL FORCE

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Abstract

This research is about the simulation of a mechanical system consisting of a mass, a spring, a shock absorber and the application of an external force, allows to understand the analytical and numerical behavior of the differential equation that governs said event. In this analysis the differential equation is obtained and solved analytically and numerically, for the numerical solution Simulink was used, since this allows the design and simulation of a system of textual and graphic form, which are fundamental elements for programming and operation of a system.

Keywords: mechanical system, differential equation, simulation.

1. SIMULATION OF A MASS-SPRING-SHOCK MECHANICAL SYSTEM WITH VARIABLE EXTERNAL FORCE.

Consider the dynamic system composed of the elements: body mass, elastic force or spring, damping force and a variable external force. See the following figure.

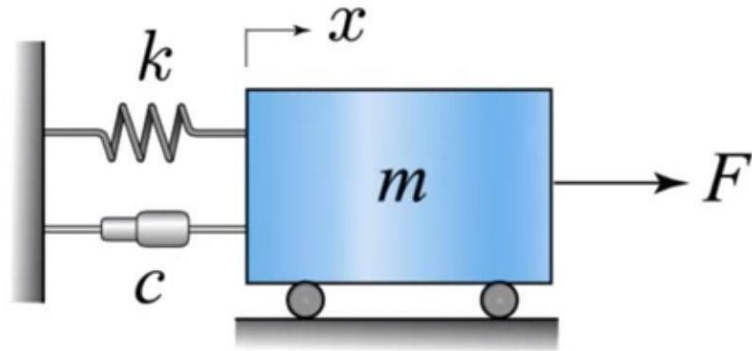


Figure 1. Mechanical System: Mass-Spring-Shock and External Force

To analyze this dynamic system it is necessary to perform the following steps:

- A. Qualitative study of the laws that govern the system.
- B. Differential equation that governs the system.
- C. Solution of the differential equation.
- D. Design the system in simulink.
- E. Simulate the system with the following information:
 - a. $K = 1$, $c = 1$, $m=1$ and $F= \text{sen}(t)$

A. Qualitative study of the laws that govern the system

The free body diagram is:

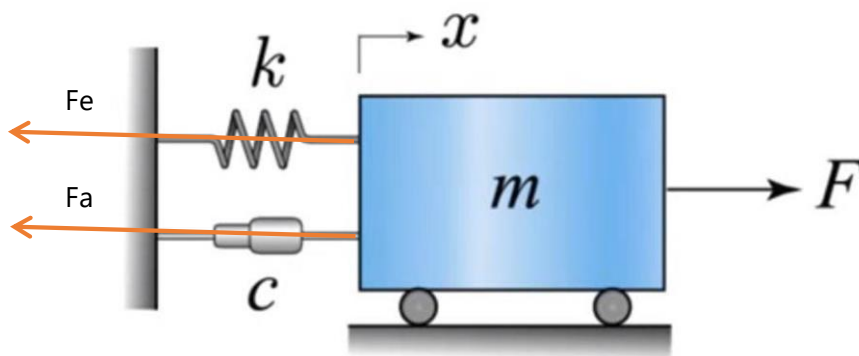


Figura 2. Diagram of mechanical system forces.

Where:

F_e It is the elastic force of the spring, given by Hooke's Law.

$$F_e = -k x$$

F_a Is the force of the shock absorber that is given by:

$$F_a = -c v$$

F It is the external force given by:

Adding the force vectors we have:

$$\sum F_T = F_a + F_e + F \quad (\text{ec. 1})$$

As $\sum F_T = m a$

$$m a = -c v - k x + F$$

$$m a + c v + k x = F \quad (\text{ec. 2})$$

B. Differential equation that governs the system

It can be seen that the time dependent variables are:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = \frac{dx}{dt}$$

$$F = F(t)$$

Substituting in ec. two.

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + k x = F \quad (\text{ec. 3})$$

It is the differential equation that governs the system.

C. Solution of the differential equation

To solve equation 3, the following solution is proposed:

$$x(t) = x(t)_h + x(t)_p$$

Where:

$x(t)_h$ is the homogeneous solution.

$x(t)_p$ is the particular solution.

Solving we have the homogeneous equation we have:

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + k x = 0$$

$$x(t)_h = c_1 e^{-\frac{1}{2} \frac{c - (c^2 - 4 k m)^{\frac{1}{2}}}{m} t} + c_2 e^{-\frac{1}{2} \frac{c + (c^2 - 4 k m)^{\frac{1}{2}}}{m} t}$$

Solving we have the non-homogeneous equation for the case $F(t) = \sin(t)$ we have:

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + k x = \text{sen}(t)$$

We propose to

$$x(t)_p = A \sin(t) + B \cos(t)$$

$$x(t)_p = \frac{k - m}{c^2 + k^2 - 2 k m + m^2} \sin(t) + \frac{c}{c^2 + k^2 - 2 k m + m^2} \cos(t)$$

Then the general solution of ec. 3 is:

$$x(t) = c_1 e^{-\frac{1}{2} \frac{c - (c^2 - 4 k m)^{\frac{1}{2}}}{m} t} + c_2 e^{-\frac{1}{2} \frac{c + (c^2 - 4 k m)^{\frac{1}{2}}}{m} t}$$

$$+ \frac{k - m}{c^2 + k^2 - 2 k m + m^2} \sin(t) + \frac{c}{c^2 + k^2 - 2 k m + m^2} \cos(t)$$

D. Design the system in simulink

We open Matlab, and on the command line we call simulink

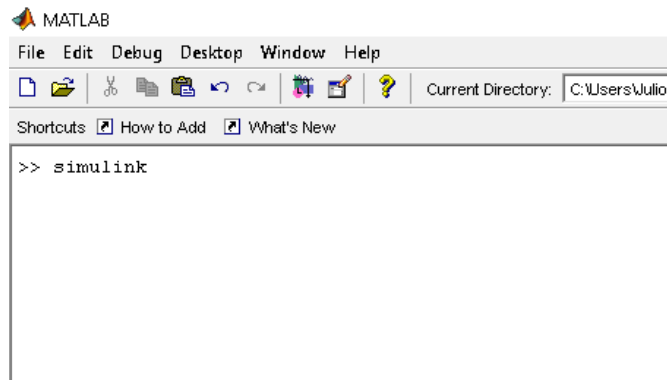


Figure 3. Matlab environment.

It shows us the simulink library

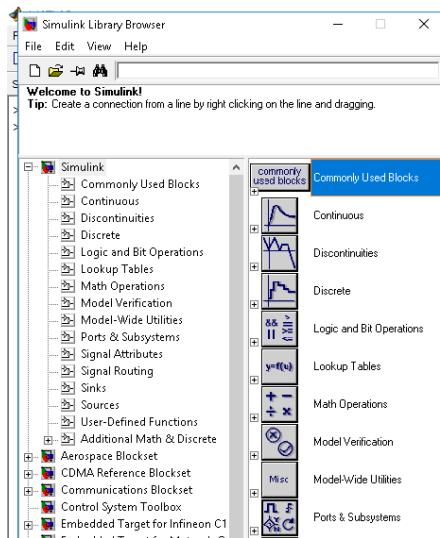


Figura 4. Simulink library.

We click again, and the simulation window appears

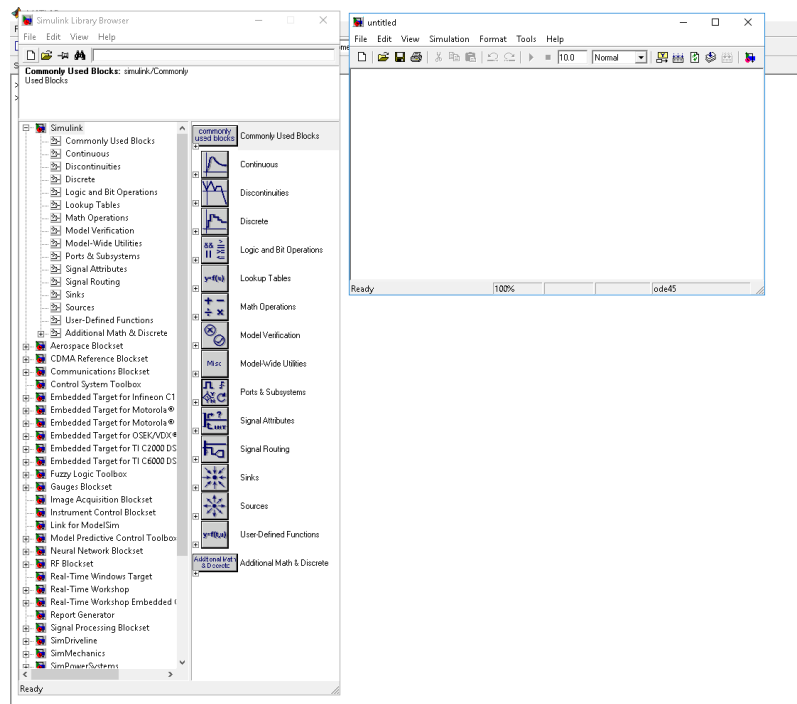


Figure 5. Simulink simulation window.

We look for the elements we need to simulate the EDO

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + k x = sen(t)$$

We clear the higher order derivative, this is:

$$\frac{d^2x}{dt^2} = \frac{1}{m} \left(-c \frac{dx}{dt} - k x + sen(t) \right)$$

We can see that the equation to be simulated requires the following objects

- An adder that will contain (Two subtractions and one sum).
- Three constant multipliers or amplifiers.
- A sine shaped external source.

We look for the source. We drag the source to the simulation window

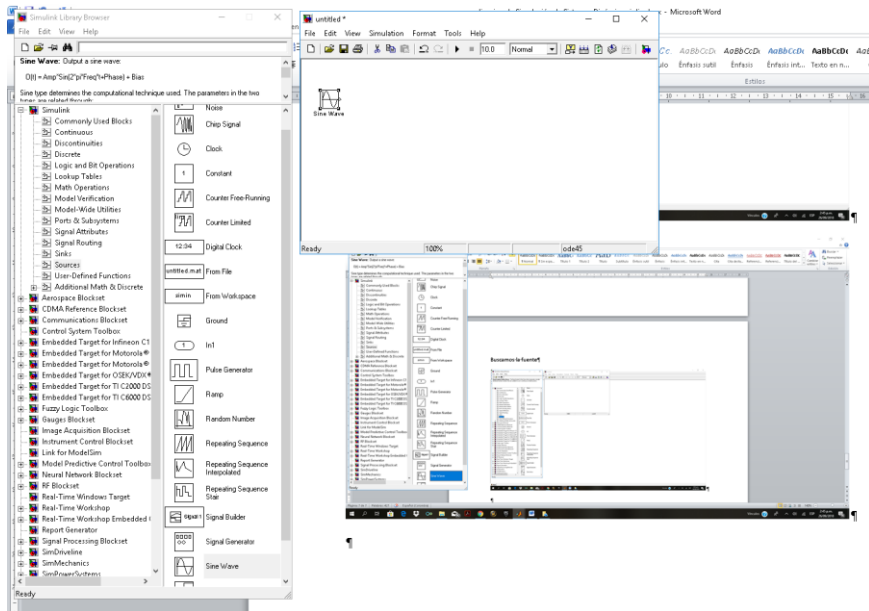


Figure 6. Power source in simulink simulator.

We look for an output element, such as the oscilloscope, we click sinks. We select the oscilloscope. We drag the oscilloscope to the simulator

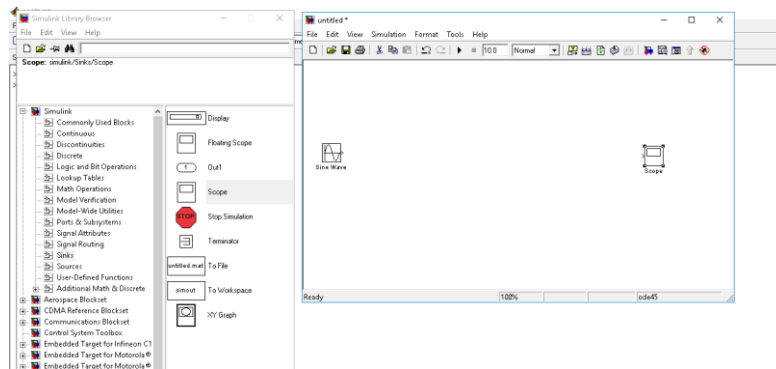


Figure 7. Oscilloscope in simulink simulator.

We look for a summed block, for this we go to the menu of mathematical operators. We select the adder. We drag the adder to the simulator

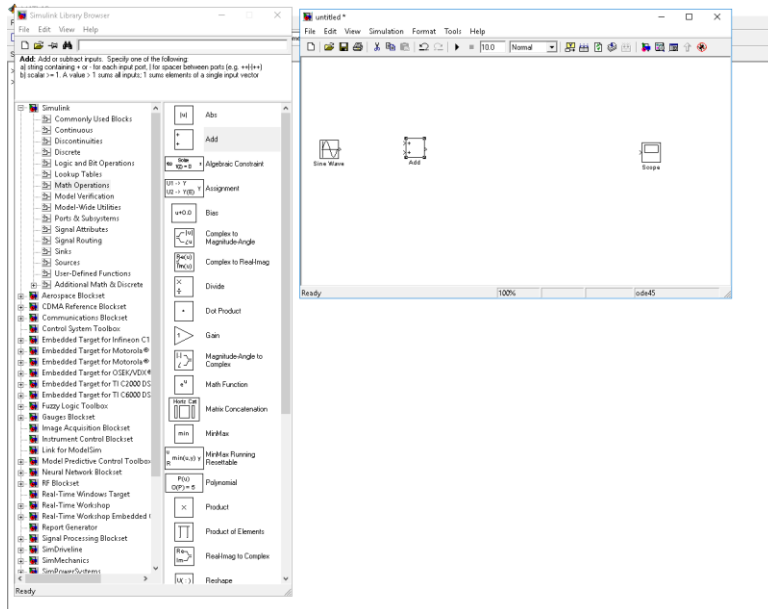


Figure 8. Adder in the simulink simulator.

We do two clicks on add. We program the adder for our system. We give ok to the dialog box of the adder or add and we have the form.

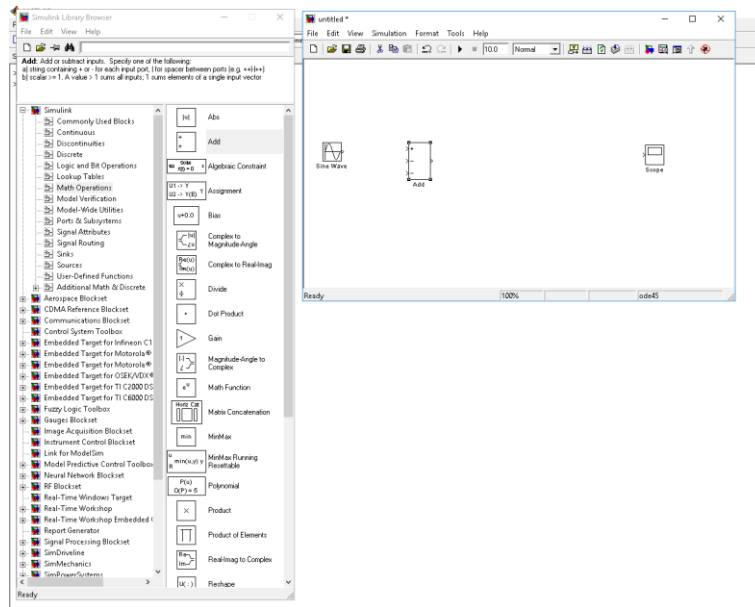


Figure 9. Expand the operations in the adder in the simulink simulator.

As the equation is second order we need two continuous integrators, for this we go to the continuous menu. Select the integrators and drag them to the simulator.

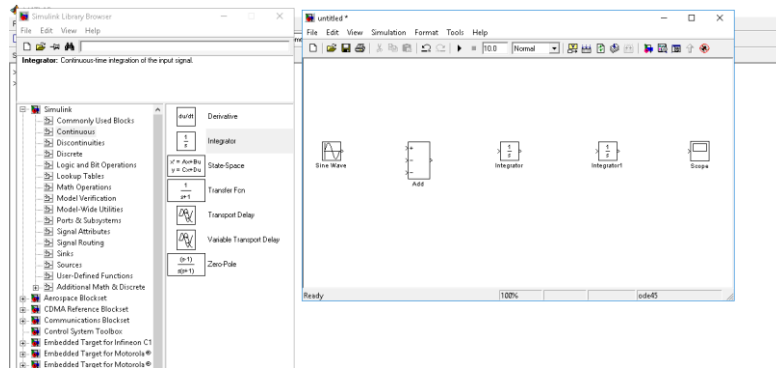


Figure 10. Integrator in simulink simulator.

We join the elements to identify the variables, for this we click at the beginning and leave it held until we reach the end of each element.

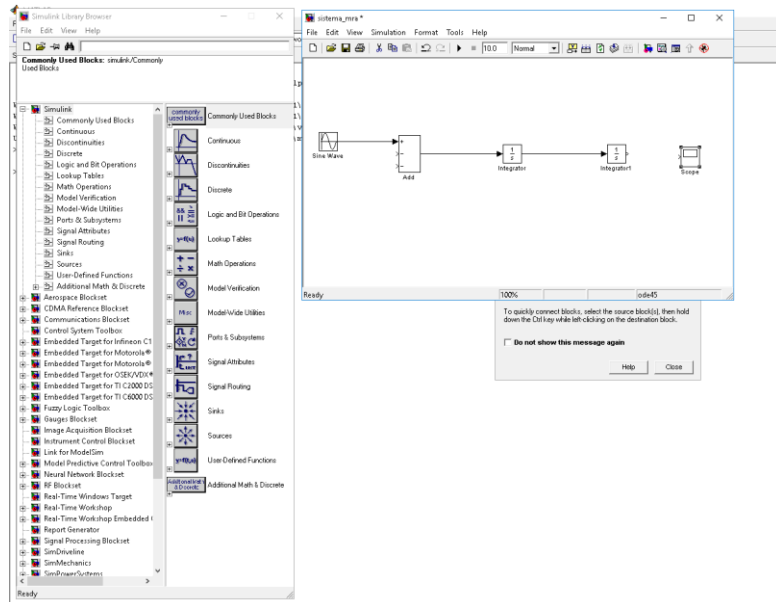


Figure 11. Joining the logical elements in the simulink simulator.

We identify the variables, for that we make two clicks near the signal lines.

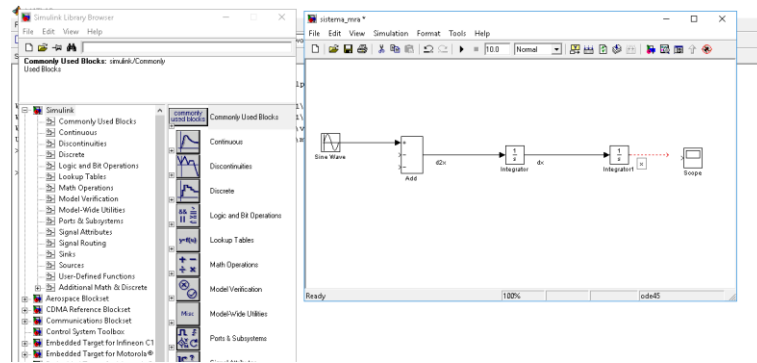


Figure 12. Identification of the variables in the simulink simulator.

We connect the variables with the adder.

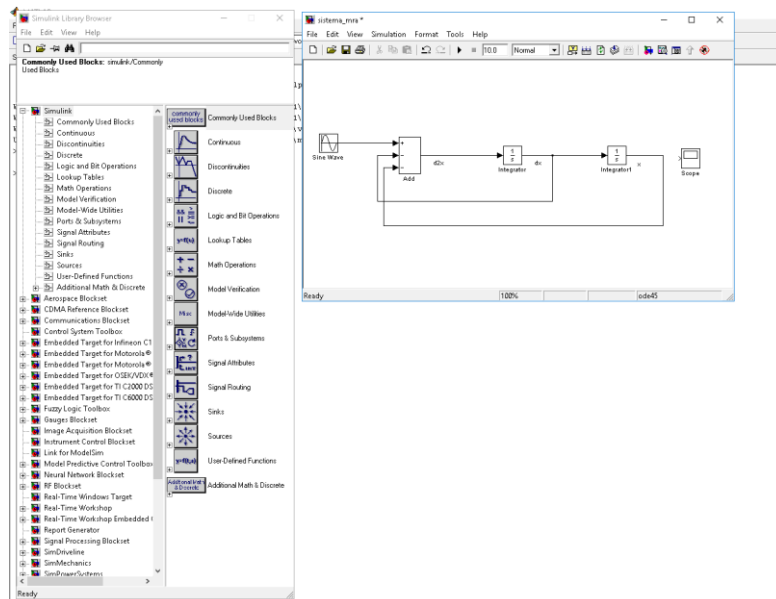


Figure 13. Union of the variables in the simulink simulator.

We multiply each output by its factor, for this we go menu of mathematical operators. We choose the profit one. We drag, until we get to the line we have a factor. We identify each factor by clicking on its name.

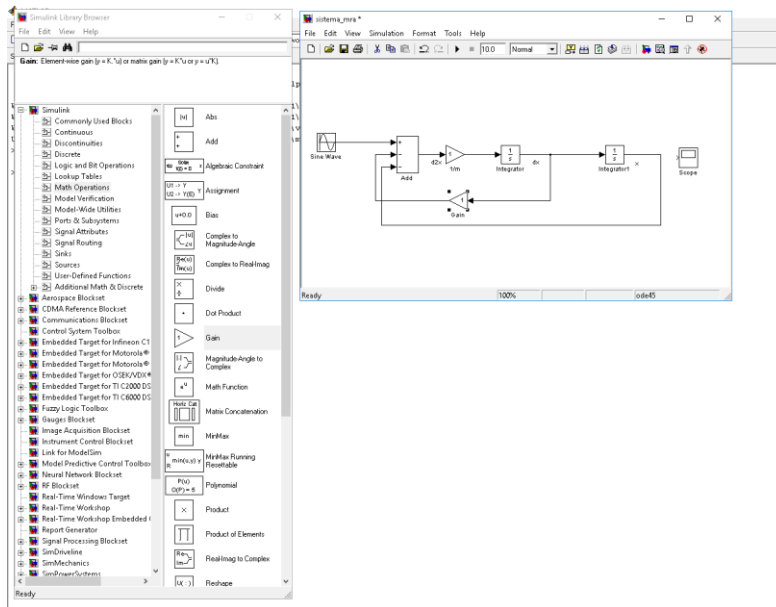


Figure 14. The multiplier in the simulink simulator.

We identify the speed factor.

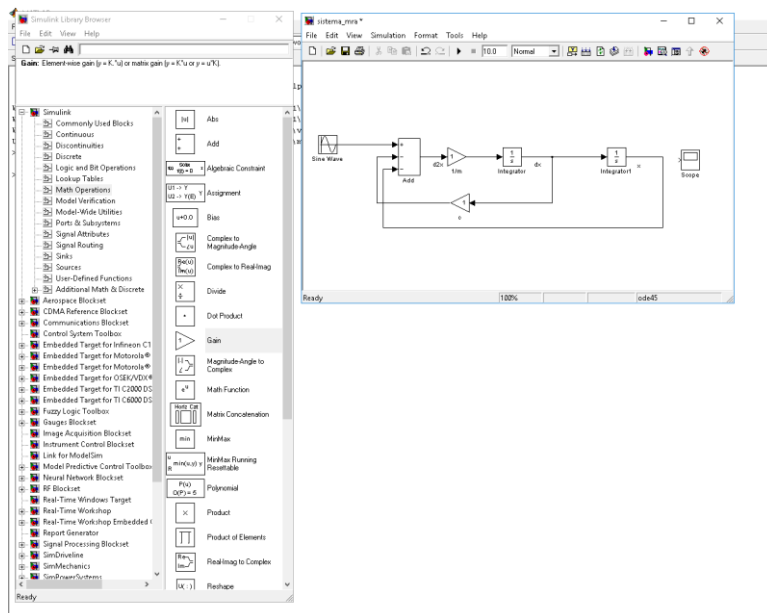


Figure 15. Identification of the speed variable in the simulink simulator.

For the line of displacement or x the factor is. We identify the displacement factor.

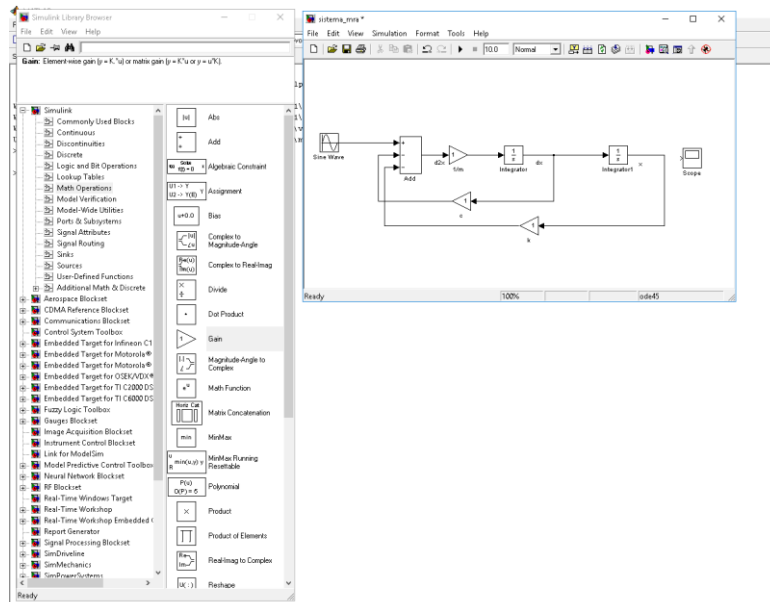


Figure 16. Identification of the displacement variable in the simulink simulator.

We can appreciate that the constant of the factor of each line has the value of 1. We connect the oscilloscope to see the outputs of the simulator.

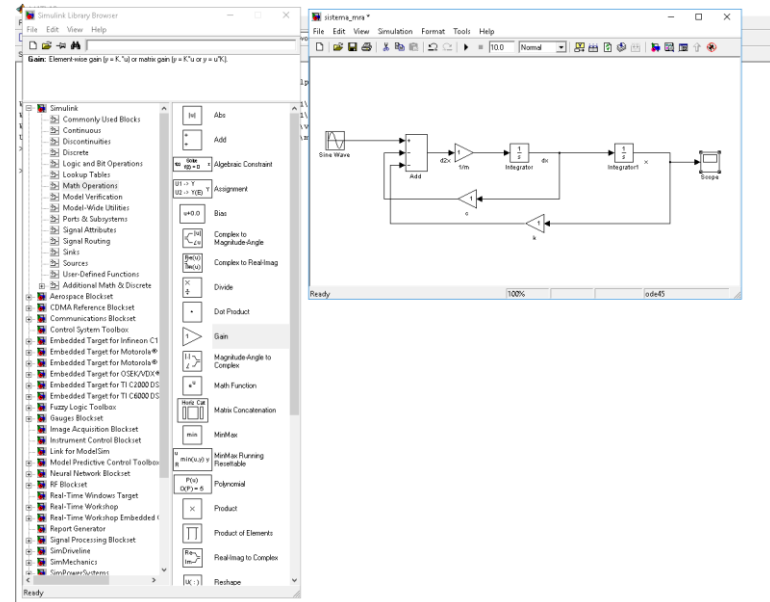


Figure 17. Value of the constants in each model line in the simulink simulator.

E. Now that the simulator is ready, it will start running for each case::

a) Simulate the system with the following information:

- a. $K = 1$, $c = 1$, $m=1$ and $F = \text{sen}(t)$

Solution:

We configure the constants, making two clicks on each factor and giving the values they indicate.

The constant mass, $m = 1$.

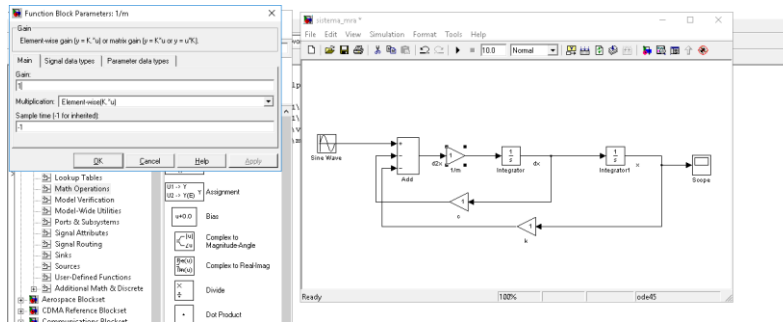


Figure 18. Assignment of $m = 1$ in the simulink simulator.

The buffer constant, $c = 1$.

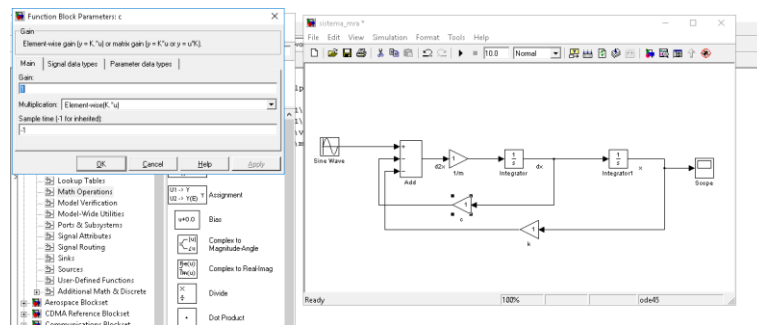


Figure 19. Assignment of $c = 1$ in the simulink simulator..

The spring constant, $k = 1$.

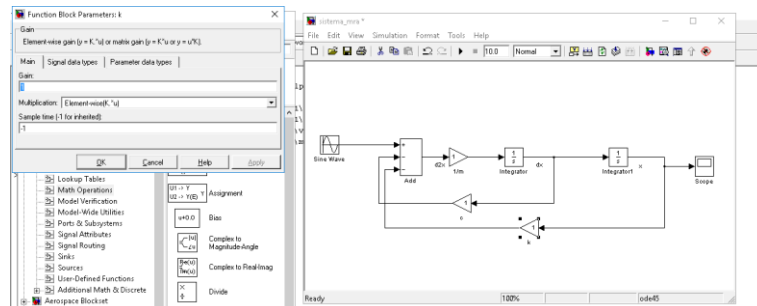


Figure 20. Assignment of $k = 1$ in the simulink simulator.

The external source, which is: $F = \sin(t)$, which has an amplitude of 1 and a frequency of 1. Making two clicks on the source and changing the parameters we have:

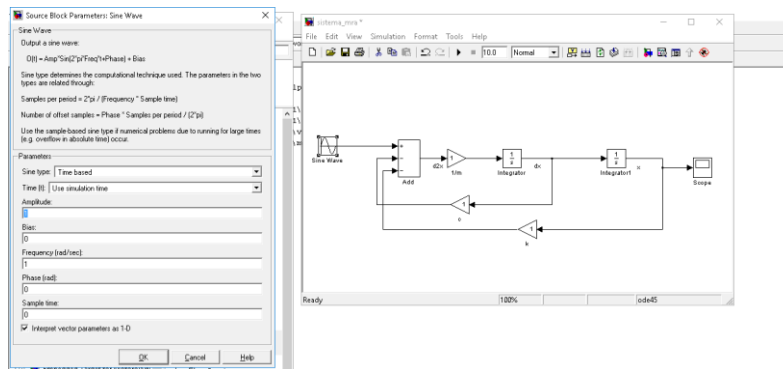


Figure 21. Assignment of the equal amplitude 1 in the power source in the simulink simulator.

We run the simulation.

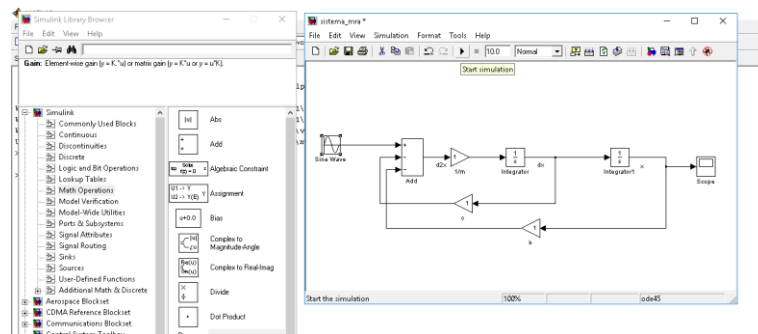


Figura 22. Run the simulation in simulink.

We give two clicks on the oscilloscope and observe the results:

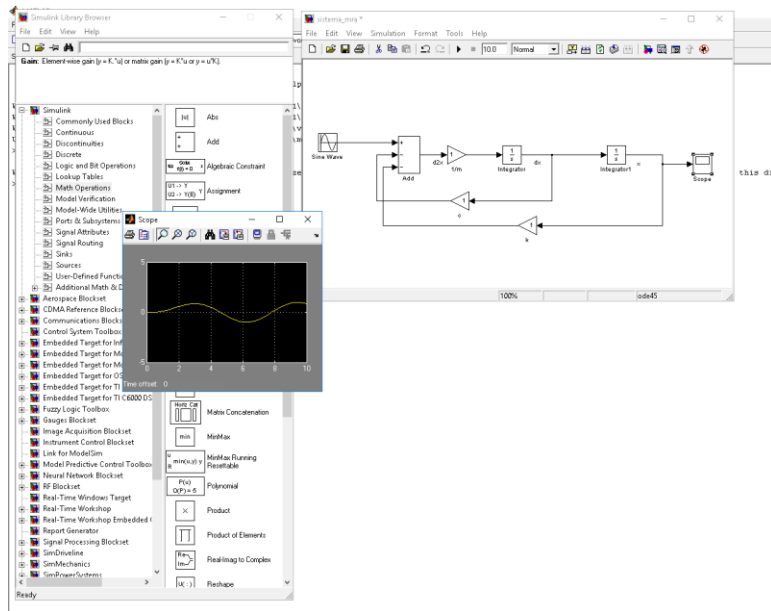


Figura 23. Activation of the oscilloscope in the simulink simulator.

Expand the oscilloscope to appreciate the system output function.

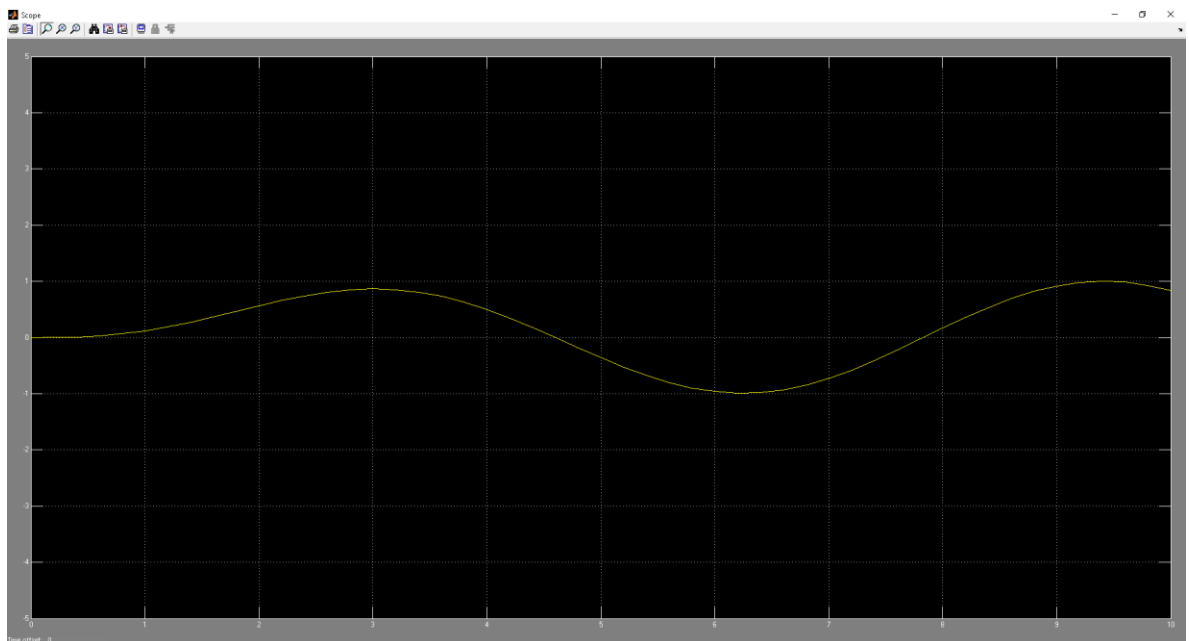


Figure 24. The oscilloscope in the simulink simulator.

By default he takes the time of [0, 10] by jumping 0.2.

Now we ask for the output data in the form of a table: We click on the oscilloscope parameters button.

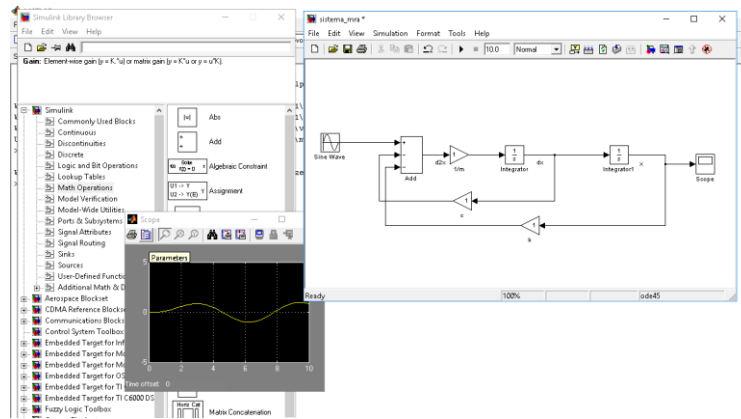


Figure 25. The parameter option in the simulink simulator oscilloscope.

They go to the data history tab.

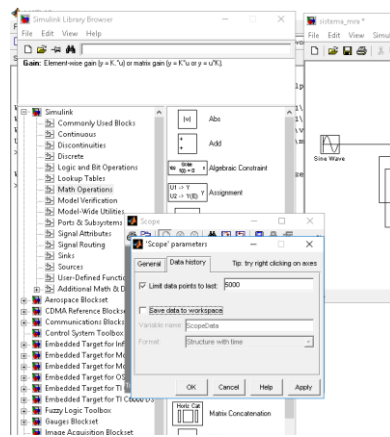


Figure 26. Windows of the parameter option in the simulink simulator oscilloscope.

Enable save to workspace.

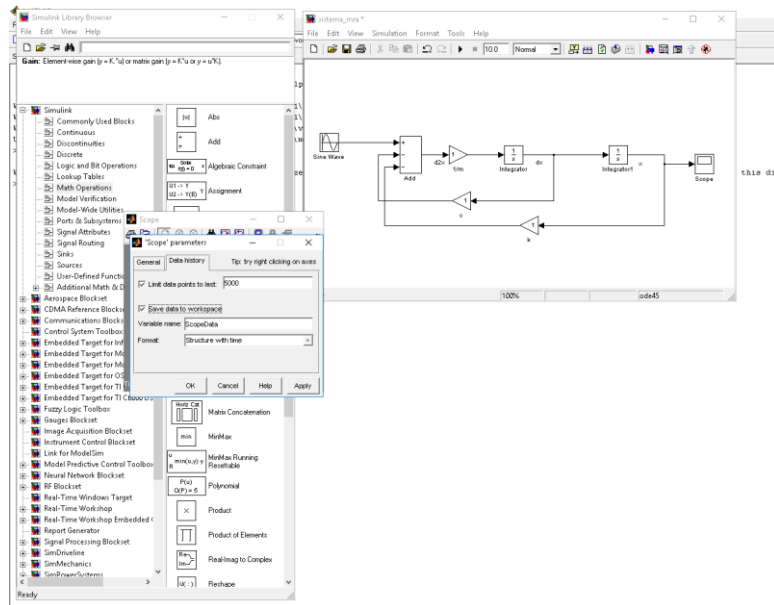


Figure 27. Workspace activation in the simulink simulator oscilloscope parameter window.

They run the simulation again, and ask to see the workspace window.

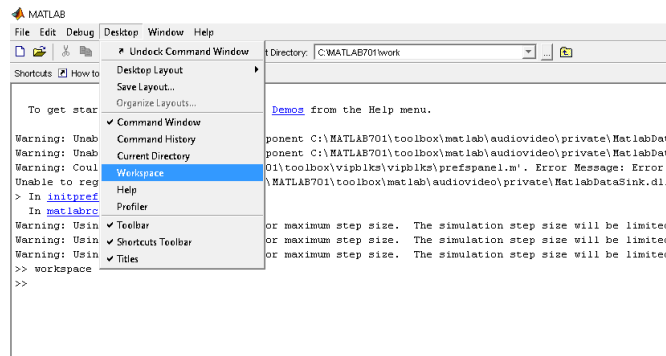


Figure 28. Show the workspace window in the simulink simulator.

there are two clicks on the Scopedata variable.

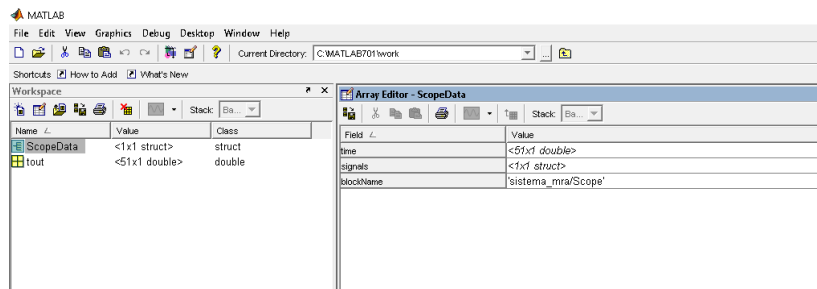


Figure 29. Simulink simulator oscilloscope data window.

Click on signals.

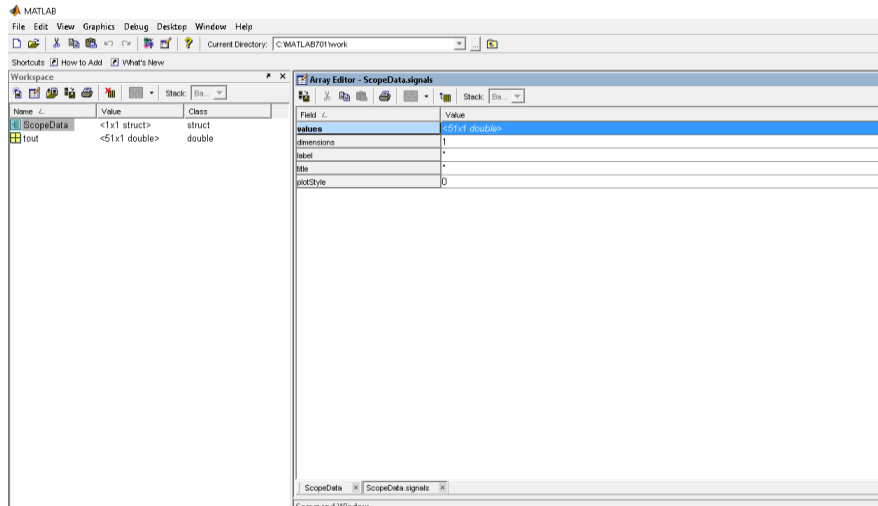


Figure 30. Data from simulink simulator oscilloscope signals.

And then about signal values.

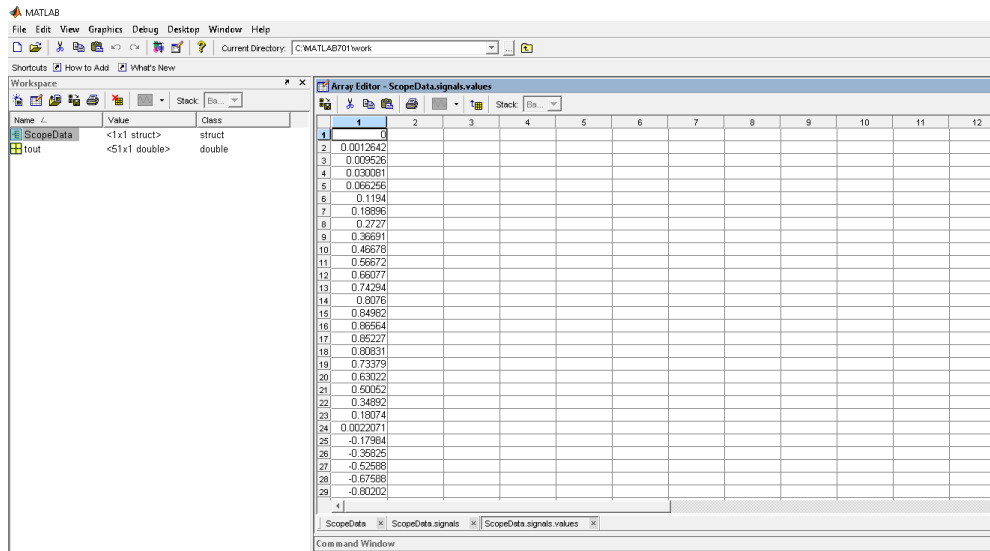


Figure 31. Simulink simulator oscilloscope data values.

Time values are appreciated when they click on the tout variable in the workspace window.

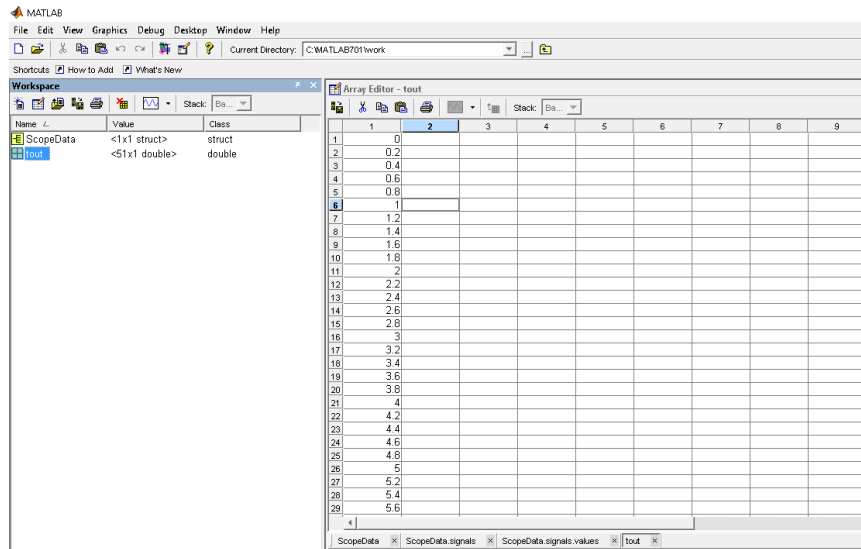


Figure 32. Time values in the simulink simulator oscilloscope.

To generate a data of the previous information we generate two vectors, the time and the displacement, this is done as follows:

Select the weather data and copy it wherever you want (Excel, Word, or Matlab).

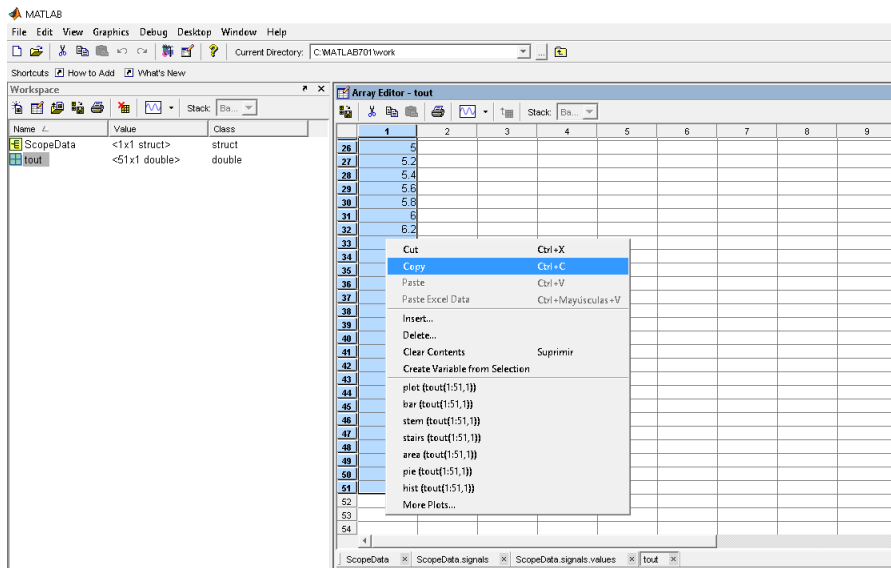


Figure 33. Export time data from the simulink simulator oscilloscope.

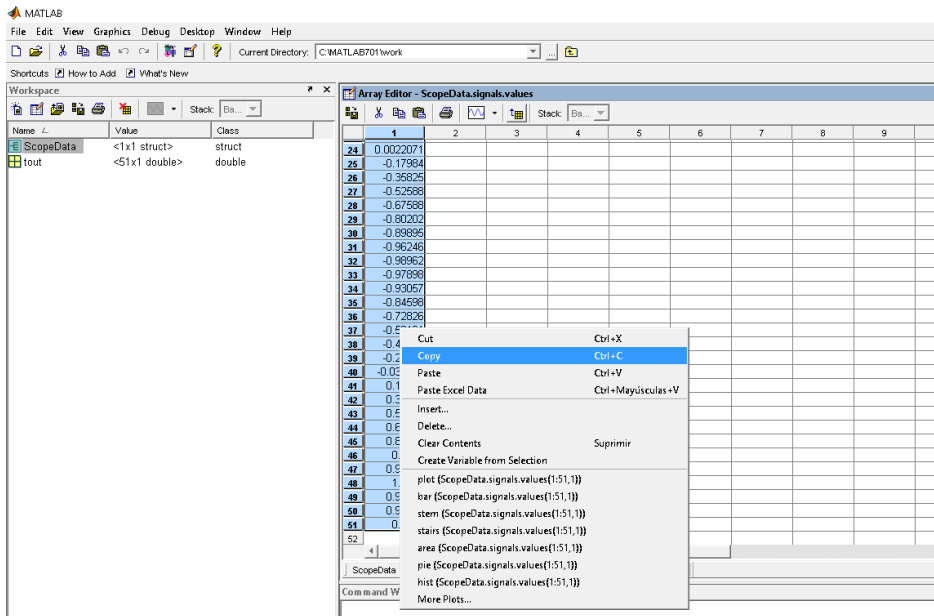


Figure 34. Export displacement data from simulink simulator oscilloscope.

Data sent to Excel.

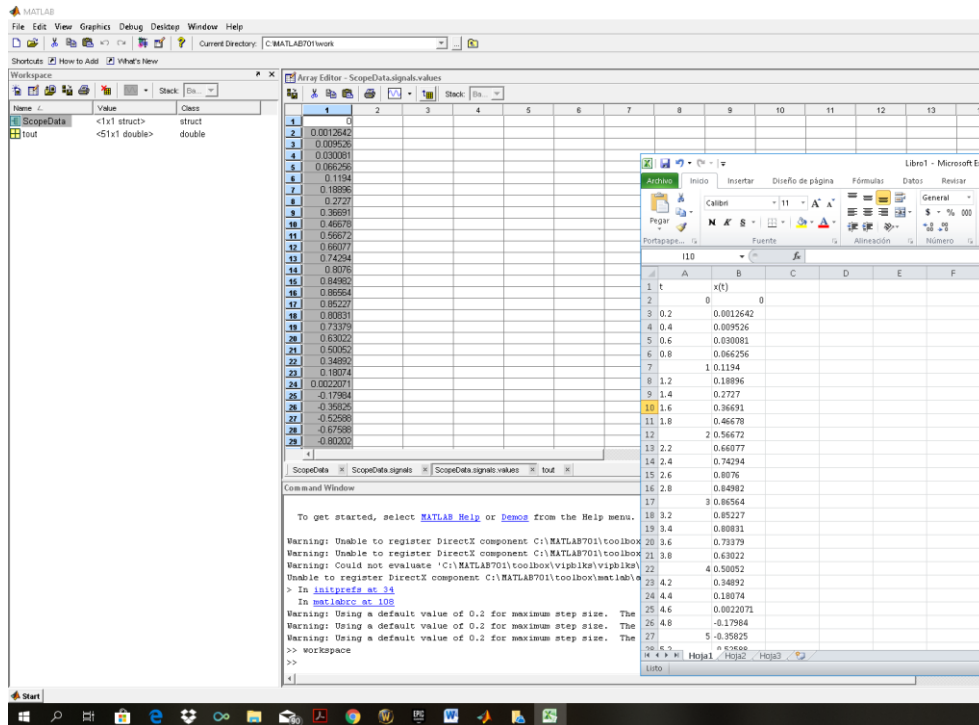


Figure 35. Export time and displacement data from simulink simulator to excel oscilloscope.

2. ANALYSIS AND RESULTS OF THE SIMULATION

Based on the simulation developed for the forced mechanical system, the following information was obtained: graph of the variables time and displacement of the system and data of the points $(t, x(t))$ of the simulation.

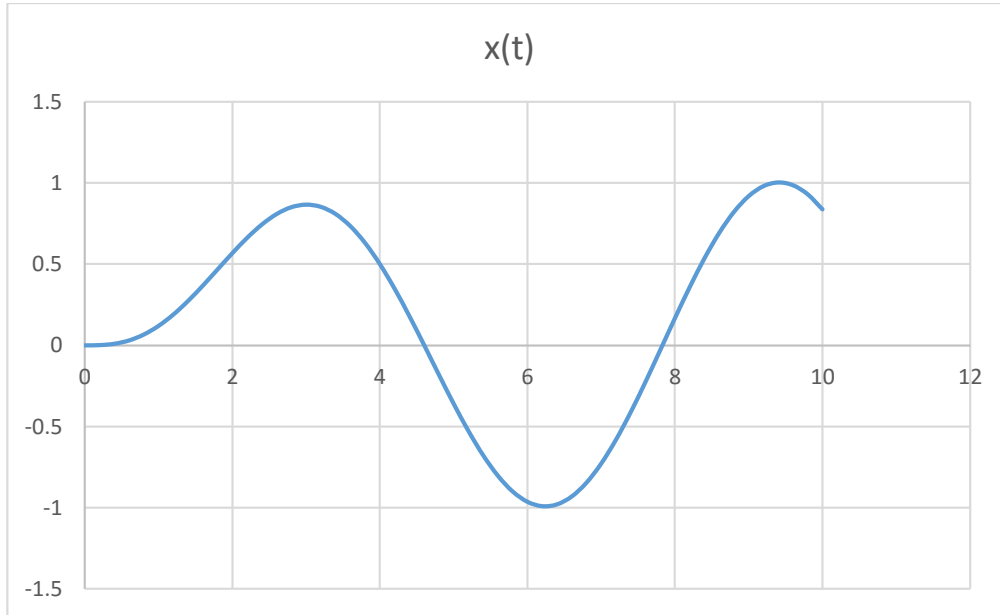


Figure 36. Graph of t vs x

Time (t)	Displacement $x(t)$
0	0
0,2	0,0012642
0,4	0,009526
0,6	0,030081
0,8	0,066256
1	0,1194
1,2	0,18896
1,4	0,2727
1,6	0,36691
1,8	0,46678
2	0,56672
2,2	0,66077
2,4	0,74294
2,6	0,8076
2,8	0,84982
3	0,86564
3,2	0,85227
3,4	0,80831

3,6	0,73379
3,8	0,63022
4	0,50052
4,2	0,34892
4,4	0,18074
4,6	0,0022071
4,8	-0,17984
5	-0,35825
5,2	-0,52588
5,4	-0,67588
5,6	-0,80202
5,8	-0,89895
6	-0,96246
6,2	-0,98962
6,4	-0,97898
6,6	-0,93057
6,8	-0,84598
7	-0,72826
7,2	-0,58181
7,4	-0,41223
7,6	-0,22608
7,8	-0,030614
8	0,16649
8,2	0,35747
8,4	0,53474
8,6	0,69128
8,8	0,82082
9	0,9182
9,2	0,97947
9,4	1,0022
9,6	0,98529
9,8	0,92949
10	0,8369

These are the results of the displacement of the mechanical system governed by the differential equation $m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + k x = F$ when analyzing the time variable in the interval $[0, 10]$ with a jump of $h = 0.2$ and with the following values in its parameters:

- Spring constant: $k = 1$.
- Damper constant: $c = 1$.
- Body mass: $m = 1$

- External force: $F = \sin(t)$

It is important to highlight that this simulation allows to appreciate the behavior of the system based on the values that we assign both to the parameters and in the time interval where you want to analyze the system.

3. CONCLUSIONS

To perform real-time simulations of systems of differential equations that govern a physical or chemical phenomenon, it can be modeled and implemented with the Simulink simulator, which is a tool that brings the Matlab programming language. In addition, Simulink has a series of mathematical functions, logical blocks, sources and connectors that allow visual programming in real time. It is for these reasons that the simulations of dynamic systems and the design of models are very worked in Simulink, since this provides a diversity of facilities and options for the programmer or the researcher such as:

- Have a graphic editor for the construction and manipulation of block diagrams.
- Have a library of predefined blocks for modeling continuous and discrete time systems.
- Solvers of ordinary and partial differential equations with fixed and variable pitch size.
- Output screens for the oscilloscope and data capture (scope and data display) for displaying results.
- Project tools and data management for handling model and data files.
- Model analysis tools to refine your architecture and increase simulation speed.
- Matlab function block for the import of Matlab algorithms into the models.
- Legacy code tool to import C and C ++ code into models.

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