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Quadratic Modelling of Uncertainty Costs for Renewable Generation and its Application on Economic Dispatch

Modelación Cuadrática de Costos de Incertidumbre para Generación Renovable y su aplicación en el Despacho Económico

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Abstract

This article presents a method for the inclusion in the economic dispatch of renewable energy generation plants. It is used uncertainty costs, which are inherent to the stochastic nature of their primary energy source. In order to reach this objective, it is taken several investigations that have been carried out the modeling of this cost of uncertainty and from these, to achieve a quadratic approximation that adapts to the classical cost functions used in the economic dispatch models.

Keywords:
Wind power generation; Photovoltaic generation; Small hydroelectric plants; Quadratic approximation, Uncertainty cost; Economic dispatch.

Resumen

Este artículo presenta un método para la inclusión en el despacho económico de plantas de generación de energía renovables. Para ello se utilizan costos de incertidumbre, los cuales son inherentes a la naturaleza estocástica de su fuente primaria de energía. Se toma como objeto de estudio diversas investigaciones que se han realizado con el fin de lograr el modelamiento de dicho costo de incertidumbre y a partir de allí, lograr una aproximación cuadrática que se adapte a las funciones de costo utilizadas clásicamente en los modelos de despacho económico.

Palabras claves:
Generación eólica; Generación solar, Pequeñas centrales hidróeléctricas; Aproximación cuadrática, Costo de incertidumbre; Despacho económico.

1. Introduction

The entrance of the renewable energies in the generation matrices of the electrical systems of the world, makes it necessary to include them in the economic dispatch exercises. Commonly, the kilowatt price of a plant's supply is determined by the costs of administration, operation and maintenance (AOM), investment and other environmental or tax costs. However, in renewable solar generation plants, plants or even in small hydroelectric power plants, there is an additional cost that is directly linked to the difficulty in specifying the availability of energy sources; not only because of a question of existence or not, but because of their variability [1][2][3].

This could make systems where the traditional economic dispatch is used, these plants are not included in the daily program and also are not part of energy contracts either short or long term; which implies the non-presence of guaranteed income to the project, making the economic recovery of the investment not entirely secure [3].

All this generates an environment of little market competitiveness that directly affects the decisions of the investment groups. This is compounded by the impossibility of participating in models such as the Reliability Charge in Colombian regulation, given the impossibility of defining a firm amount of energy.

However, this stochastic behavior can be described and modeled by probability distribution functions, as shown by [1] [2] [4] [5]. with which it is possible to obtain uncertainty costs, therefore, the possibility of participating or introducing these plants in classic economic dispatch models is opened.

For the plants, the wind speed can be modeled by the probability distribution of Weibull [1] [2]; in terms of solar plants, the irradiance depends on the geographical location in which it is located [4] [5] and also the probability distribution function can be modeled for a distribution Beta, Weibull or a distribution Log-normal [4] [**?**]. Finally, the behavior of the river flow for the Small Hydroelectric Power Plants (PCH) can be modeled by the distribution of Gumbel [7][8][9].

By obtaining a model of the behavior of the availability of the primary resources for these technologies, it is possible to program a quantity of power. However, the stochastic generator may have differences between the real power dispatched ($W_{av,i}$) and the programmed power ($W_{s,i}$) by the system operator; which leads to consider the cost of uncertainty either by concepts of underestimation ($W_{s,i} < W_{av,i}$) or overestimation ($W_{s,i} > W_{av,i}$).

Based on the above, the functions of Uncertainty Cost (UFC for its acronym in English) that correspond to the sum of the costs for underestimation ($C_{u,i}$) and the costs for overestimation ($C_{o,i}$) in the following way:

$$UCF = C_{u,i}(W_{s,i}, W_{av,i}) + C_{o,i}(W_{s,i}, W_{av,i}) \tag{1}$$

Thus, throughout this article different scenarios for obtaining cost functions of uncertainty and simulation are presented in economic dispatch models of generation plants. photo voltaic, wind and small hydroelectric power.

2. Nature of the Cost of Uncertainty function

The uncertainty about the instantaneous power availability that characterize the solar energy sources, and even in the small hydroelectric power stations; On the electric power systems that bet on its incorporation, they produce a change in the way in which the economic dispatch models are used, in such a way, that the use of stochastic models. Through this section we will obtain the analytically functions of the cost of uncertainty for energy, solar, and small hydroelectric power plants; through the mathematical formulation of the expected value of said cost, considering distribution functions of probability for each of the primary sources. The validation of the same is done through Monte Carlo simulations [3].

The main uses of Monte Carlo simulation are the modeling of physical systems or stochastic processes that are composed of random variables, as long as it has probability density functions [11]. Also, this simulation is used to study the behavior of complex non-deterministic systems, computationally generating a large number of random values and thus predicting the behavior of the system.

As mentioned in the previous section, each uncertainty cost function found is composed of two parts: Cost for underestimation ($C_{u,i}$) and the costs for overestimation ($C_{o,i}$). It is the quadratic modeling of these functions, however, the one that generates the best conditions for the inclusion of these sources in the economic dispatch models, converting them as follows. in the main contribution and objective of this article.

3. Solar energy uncertainty cost function

As is well known, the primary energy in this technology is irradiance, which depends directly on the environmental conditions in which the plant is located. For this article it is assumed that the solar irradiance behaves like a Log-normal distribution, that although it does not represent a general model for any latitude or altitude, it can be used under specific conditions. [5] [12] [13]. Thus, the log-normal probability distribution is described by:

$$f_G(G) = \frac{1}{G\beta\sqrt{2\pi}} * \exp\left\{-\frac{[ln(G) - \lambda]^2}{2\beta^2}\right\} \tag{2}$$

where G is the solar irradiance, f_G is the corresponding probability distribution function and λ and β are the parameters of scale and location of the Log-normal distribution.

Likewise, the power generated (W_{PV}) by the solar plant as a function of the solar irradiance of the site in question is given by the following relationships, in which it is taken into account a reference value of solar irradiance (R_C) according to [4]:

- **Condition A:** for $0 < G < R_c$

$$W_{PV} = \frac{W_{PVr} G^2}{G_r R_C} \tag{3}$$

- **Condition B:** for $G > R_c$

$$W_{PV} = \frac{W_{PVr} G}{G_r} \tag{4}$$

where in ref grc- and ref grc +: G is the solar irradiance, G_r is the nominal solar irradiance of the geographical place, and W_{PVr} is the nominal active power of the photovoltaic generation source.

Under these considerations, in [6] the mathematical development of the uncertainty cost function is presented in detail both for overestimation and for underestimation for both conditions described above. In both cases a linear function is assumed to calculate the costs. However, these may change according to the methodology used.

3.1. Mathematical formulation in the underestimated condition

The formulation of the cost of uncertainty due to underestimation is presented as follows:

$$E[C_{PV,u,i}(W_{PV,s,i}, W_{Pv,i})] = \int_{W_{PV,s,i}}^{W_{PV,\infty,i}} c_{PV,u,i}(W_{PV,i} - W_{PV,s,i}) \cdot fW_{PV}(W_{PV,i}) \cdot dW_{PV,i} \tag{5}$$

where $E[C_{PV,u,i}(W_{PV,s,i}, W_{Pv,i})]$ is the expected value of the cost due to the underestimation, $c_{PV,u,i}$ is the penalty cost coefficient for underestimation, $fW_{PV}(W_{PV,i})$ is the probability distribution of the primary source, $W_{PV,\ infty,i}$ is the maximum output power of the generator, $W_{PV,s,i}$ is the power programmed by the economic dispatch model and $W_{PV,i}$ is the power available in the generator.

Finally, the underestimation cost is directly conditioned with the generated power value W_{Rc} with the irradiance R_C. The expressions for the expected cost due to the underestimation in each of the conditions are described below. [6] [3]:

- **Condition A:** for $0 < W_{PV,i} \leq W_{Rc}$:

$$\begin{aligned} E[C_{PV,u,i}(W_{PV,s,i}, W_{Pv,i}), A] &= \frac{(-1)c_{PV,u,i}W_{PV,s,i}}{2} \\ &\left(erf(\frac{(\frac{1}{2}\ln(\frac{W_{Rc}G_rR_c}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta}) - erf(\frac{(\frac{1}{2}\ln(\frac{W_{PV,s,i}G_rR_c}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta})\right) \\ &+ \frac{c_{PV,u,i}W_{PVr} \cdot e^{2\lambda+2\beta^2}}{2G_rR_c}\left(erf(\frac{(\frac{1}{2}\ln(\frac{W_{Rc}G_rR_c}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta} - \sqrt{2}\beta)\right. \\ &\left. - erf(\frac{(\frac{1}{2}\ln(\frac{W_{PV,s,i}G_rR_c}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta} - \sqrt{2}\beta)\right) \end{aligned} \tag{6}$$

- **Condition B:** for $W_{PV,i} > W_{Rc}$:

$$\begin{aligned} E[C_{PV,u,i}(W_{PV,s,i}, W_{Pv,i}), B] &= \frac{c_{PV,u,i}W_{PV,s,i}}{2} \\ &\left(erf(\frac{(\ln(\frac{W_{Rc}G_r}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta}) - erf(\frac{(\ln(\frac{W_{PV,\infty,i}G_r}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta})\right) \\ &+ \frac{c_{PV,u,i}W_{PVr} \cdot e^{\lambda+\beta^2/2}}{2G_r}\left(erf(\frac{(\ln(\frac{W_{PV,\infty,i}G_r}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta} - \frac{\beta}{\sqrt{2}})\right. \\ &\left. - erf(\frac{(\ln(\frac{W_{Rc}G_r}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta} - \frac{\beta}{\sqrt{2}})\right) \end{aligned} \tag{7}$$

3.2. Mathematical formulation in the overestimated condition

The formulation of the cost of uncertainty due to overestimation is presented as follows:

$$E[C_{PV,o,i}(W_{PV,s,i}, W_{Pv,i})] = \int_{0}^{W_{PV,s,i}} c_{PV,o,i}(W_{PV,s,i} - W_{PV,i}) \cdot fW_{PV}(W_{PV}) \cdot dW_{PV,i} \tag{8}$$

where $E[C_{PV,or,i}(W_{PV,s,i}, W_{Pv,i})]$ is the expected value of the cost due to the overestimation, $c_{PV,or,i}$ is the penalty cost coefficient for overestimation, $fW_{PV}(W_{PV})$ is the probability distribution of the primary source, $W_{PV,s,i}$ is the power programmed by the economic dispatch model and $W_{PV,i}$ is the power available in the generator [3].

Finally, the cost for overestimation is directly conditioned with the generated power value W_{Rc} with the irradiance R_C. The expressions for the expected cost due to the overestimation in each of the conditions are described below. [6]:

- **Condition A:** for $0 < W_{PV,i} \leq W_{Rc}$:

$$E[C_{PV,o,i}(W_{PV,s,i}, W_{PV,i}), A] = \frac{-c_{PV,O,i}W_{PV,s,i}}{2} \left(1 + erf(\frac{(\frac{1}{2}\ln(\frac{W_{Rc}G_rR_c}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta})\right) + \frac{c_{PV,O,i}W_{PVr} \cdot e^{2\lambda+2\beta^2}}{2G_rR_c} \left(erf(\frac{(\frac{1}{2}\ln(\frac{W_{Rc}G_rR_c}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta} - \sqrt{2}\beta) + 1\right) \tag{9}$$

- **Condition B:** for $W_{PV,i} > W_{Rc}$:

$$E[C_{PV,O,i}(W_{PV,s,i}, W_{Pv,i}), B] = \frac{c_{PV,O,i}W_{PV,s,i}}{2} \left(erf(\frac{(\ln(\frac{W_{Rc}G_r}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta}) - erf(\frac{(\ln(\frac{W_{PV,s,i}G_r}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta})\right) + \frac{c_{PV,o,i}W_{PVr} \cdot e^{\lambda+\beta^2/2}}{2G_r}\left(erf(\frac{(\ln(\frac{W_{PV,\infty,i}G_r}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta} - \frac{\beta}{\sqrt{2}}) - erf(\frac{(\ln(\frac{W_{Rc}G_r}{W_{PVr}}) - \lambda)}{\sqrt{2}\beta} - \frac{\beta}{\sqrt{2}})\right) \tag{10}$$

Therefore, the Cost of Uncertainty Function for the solar energy plant is the result of the sum of the equations 6, 7, 9 y 10.

3.3. Monte Carlo simulation

Once the analytical formula of the cost of uncertainty is counted, the values of said cost can be obtained for any power programmed in the Economic Dispatch. For this, the Monte Carlo simulation is used, which has shown good accuracy in [6] and [10]. The main steps to carry out the simulation are:

1. A power value is set representing the power of the photovoltaic generator i programmed by the economic dispatch model ($W_{PV,s,i}$). This value is varied to know the behavior of the Cost of uncertainty with respect to the installed power.
2. A Monte Carlo scenario is generated through a random irradiance value generated for the generator $i(G_i)$ according to the Log-normal probability distribution.
3. Given the random irradiance generated, the available or generated power $W_{PV,i}$, according to the equations, is evaluated. 3 and 4.

4. In this Monte Carlo scenario the cost is evaluated: if $W_{PV,s,i} < W_{PV,i}$ then use the equation of the underestimated condition 6 or 7, and if $W_{PV,i} < W_{PV,s,i}$ then the equation is used. 'on the condition overestimated 9 or 10.
5. It is repeated from step 2 to 4 of this list for a certain number of Monte Carlo scenarios.
6. The expected value of the total accumulated cost is calculated; this amount is the expected value of the uncertainty cost function.
7. It is repeated from step 1 to 6 for each possible power value programmed by the economic dispatch model ($W_{PV,s,i}$).

The data for the realization of this study are presented in the Table 1 [6].

Income data		
Symbol	Parameter	Value
W_{PVr}	Nominal power of the generator i [MW]	65
G_r	nominal irradiance of the geographical location $[W/m^2]$	1000
R_c	Reference value of irradiance $[W/m^2]$	150
$W_{PV,\infty}$	Maximum output power	100
λ	Location parameter of the Log-normal distribution	6
β	Scale parameter of the Log-normal distribution 0.25	
N	Number of iterations	100000
$W_{PV,s,i}$	Power programmed in the generator i [MW]	
$C_{PV,u,i}$	Penalty cost coefficient due to underestimation [$/MWh]	30
$C_{PV,o,i}$	Penalty cost coefficient due to overestimation [$/MWh]	70

Table 1. Data for calculating the cost of uncertainty

Once the process is completed, the programmed power graph $W_{PV,s,i}$ is obtained with respect to the expected uncertainty cost, which is presented below. (Figure 1):

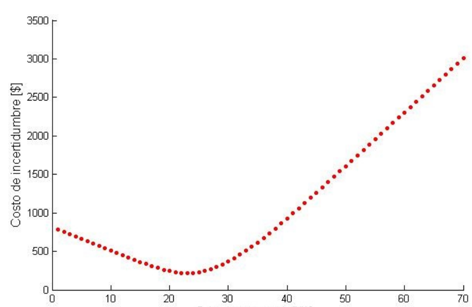


Figure 1. *Expected value of the cost of uncertainty for solar generation*

From this information, it is possible to achieve a modeling of the uncertainty cost function that has a polynomial structure taking as an independent variable the programmed power.

3.4. Polynomial and quadratic modeling of the uncertainty cost function

Through the use of numerical method tools, it is possible to obtain a polynomial function that describes in a much simpler way the behavior of the expected value of the cost of uncertainty and which, in turn, allows its inclusion in optimization simulators of power systems without having to resort to expressions such as those described in the equations 6 or 10, for quote some examples [3].

In this process it is possible to obtain functions of any degree; However, the objective of this study is to find a function of the lowest possible degree that allows the correct description of the behavior of this cost.

In this case, the degree of polynomial with which a good approximation was obtained was of degree 6. This polynomial is presented below, with the characteristic feature 11:

$$\begin{aligned} f(W_{PV,s,i}) = -4,236x10^{-}8(W_{PV,s,i})^6 + 2,833x10^{-}5(W_{PV,s,i})^5 \\ -4,501x10^{-}3(W_{PV,s,i})^4 + 0,278(W_{PV,s,i})^3 - 5,805(W_{PV,s,i})^2 \\ +11,494(W_{PV,s,i}) + 740,650 \end{aligned} \tag{11}$$

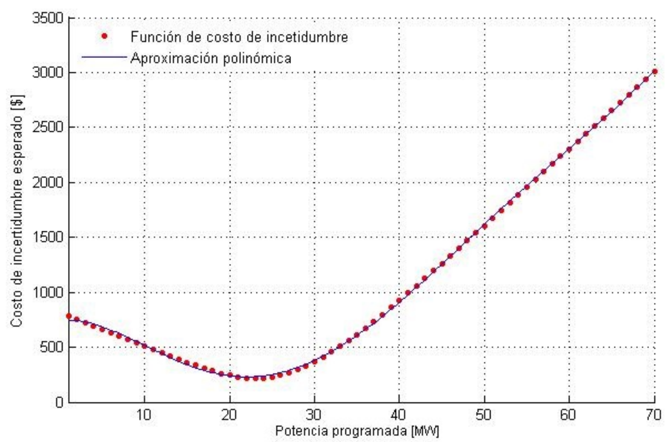


Figure 2. *Polynomial approximation of the cost of uncertainty for photovoltaic generation*

Although it is true that this function represents to a large extent the behavior of the expected value of uncertainty cost, its use in economic dispatch models may not be possible or generate many difficulties, since the functions of cost used in it are of the form of the function 34. This is why the scenario in which this plant has a power value dispatched $W_{PV,s,i}$ m 'is presented, in this way, the uncertainty cost function can be described by a quadratic function, and so be included in the economic dispatch models:

$$C_{Gi}(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i [P_{Gi}]^2 \tag{12}$$

Therefore, the quadratic function that achieves the best approximation to this new condition of the cost function of uncertainty 13 with its corresponding representation graph (Figure 3).

$$f(W_{PV,s,i}) = 0.331(W_{PV,s,i})^2 + 33.544(W_{PV,s,i}) - 918.558 \tag{13}$$

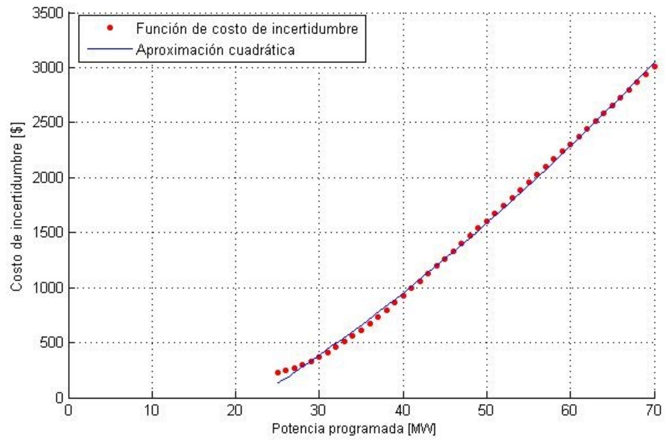


Figure 3. *Quadratic approximation of the cost of uncertainty for photovoltaic generation*

For inclusion in the economic dispatch models, this function of uncertainty costs must be accompanied by the condition 36:

$$W_{PV,s,i} \geq 25MW. \tag{14}$$

4. Uncertainties cost function generation and ethnicity

For eolic plants, the primary energy source is the wind speed (v) and depends directly on the location of the plant. However, it can be considered that its behavior follows the Rayleigh distribution function for a large number of places around the world [2] [4] [5]. Thus, the Rayleigh likelihood distribution is described by:

$$f_v(v) = \frac{v}{\sigma^2} * \exp\left\{-\left(\frac{v}{\sqrt{2}\sigma}\right)^2\right\} \tag{15}$$

where *sigma* is the scale parameter of the Rayleigh likelihood distribution function.

On the other hand, the energy conversion function has the conditions described in several investigations cite Hetzer cite Surender. 'This conversion function depends on the input cutoff speed v_i, nominal speed v_r, output cutoff speed v_o, nominal power W_r and constants ρ and κ given by:

$$\rho = \frac{W_r}{v_r - v_i} \tag{16}$$

$$\kappa = \frac{-W_r \cdot v_i}{v_r - v_i} \tag{17}$$

Thus, the conditions are:

- **Condition A:** for $v \leq v_i$ or $v \geq v_o$. In this case there is an insufficiency of energy or saturation of the generator, therefore:
$$W_w(v) = 0 \tag{18}$$
- **Condition B:** para $v_i < v < v_r$
$$W_w(v) = \rho v + \kappa \tag{19}$$
- **Condition C:** para $v_r \leq v < v_o$
$$W_w(v) = W_r \tag{20}$$

Under these considerations, [6] presents in detail the mathematical development of the cost function of uncertainty, both by overestimation and underestimation.

4.1. Mathematical formulation in the underestimated condition

The formulation of the uncertainty cost due to underestimation is presented as follows:

$$E[C_{w,u,i}(W_{w,s,i}, W_{w,i})] = \int_{W_{w,s,i}}^{W_r} c_{w,u,i}(W_{w,i} - W_{w,s,i}) \cdot fW(W_{w,i}) \cdot dW_{w,i} \tag{21}$$

where $E[C_{w,u,i}(W_{w,s,i}, W_{w,i})]$ is the expected value of the cost due to the underestimation, $c_{w,u,i}$ is the penalty cost coefficient for underestimation, $fW(W_w)$ is the probability distribution of the primary source, $W_{w,s,i}$ is the power programmed by the economic dispatch model and $W_{w,i}$ is the power available in the generator.

Finally, the expression for the expected cost due to the underestimation is described below. [6]:

$$\begin{aligned} E[C_{w,u,i}(W_{w,s,i}, W_{w,i})] = \frac{c_{w,u,i}}{2}\Bigg(&\sqrt{2}\beta\rho\sigma(erf(\frac{W_{w,s,i} - \kappa}{\sqrt{2}\beta}) \\ &-erf(\frac{W_r - \kappa}{\sqrt{2}\beta})) + 2(W_{w,s,i} - W_r)e^{-(\frac{W_r-\kappa}{\sqrt{2}\beta})^2}\Bigg) \\ &+\frac{c_{w,u,i}}{2}(e^{-\frac{v_r^2}{2\sigma^2}} - e^{-\frac{v_o^2}{2\sigma^2}})(W_r - W_{w,s,i}) \end{aligned} \tag{22}$$

4.2. Mathematical formulation in the overestimated condition

The formulation of the cost of uncertainty due to overestimation is presented as follows:

$$E[C_{w,o,i}(W_{w,s,i}, W_{w,i})] = \int_{W}^{W_{w,s,i}} c_{w,o,i}(W_{w,s,i} - W_{w,i}) \cdot fW(W_{w,i}) \cdot dW_{w,i} \tag{23}$$

where $E[C_{w,o,i}(W_{w,s,i}, W_{w,i})]$ is the expected value of the cost due to the overestimation, $c_{w,or,i}$ is the penalty cost coefficient for overestimation, $fW(W_w)$ is the probability distribution of the primary source, $W_{w,s,i}$ is the power programmed by the economic dispatch model and $W_{w,i}$ is the power available in the generator.

Finally, the expression for the expected cost due to the overestimation is described below. [6]:

$$E[C_{w,o,i}(W_{w,s,i}, W_{w,i})] = c_{w,o,i}W_{w,s,i} \cdot (1 - e^{-\frac{v_r^2}{2\sigma^2}} + e^{-\frac{v_o^2}{2\sigma^2}} + e^{-\frac{\kappa^2}{2\rho^2\sigma^2}}) - \frac{\sqrt{2\pi}C_{w,o,i}\rho\sigma}{2}\left(erf(\frac{W_{w,s,i} - \kappa}{\sqrt{2}\rho\sigma}) - erf(\frac{-\kappa}{\sqrt{2}\rho\sigma})\right) \tag{24}$$

Therefore, the Cost of Uncertainty Function for the energetic power plant is the result of the sum of the equations 22 y 24.

4.3. Monte Carlo simulation

Following the step of subsection 3.3, it is possible to obtain the respective Monte Carlo simulation. The data for the realization of this study are presented in the Table 2 [6].

Income data		
Symbol	Parameter	Value
W_{wr}	Nominal Power of generator i [MW]	150
v_i	Boot wind speed $[m/s]$	5
v_r	Rated wind speed $[m/s]$	15
v_o	Stop wind speed $[m/s]$	45
ρ	Linear constant of the power vs. speed curve $[MW/m/s]$	15
κ	Independent constant of the power vs. speed curve $[MW]$	-75
σ	Scale parameter of the Rayleigh distribution[m/s]	15.95
N	Iteration number	1000000
$W_{w,s,i}$	Power programmed in the generator i [MW]	
$C_{w,u,i}$	Penalty cost coefficient due to underestimation [$/MWh]	30
$C_{w,o,i}$	Penalty cost coefficient due to overestimation [$/MWh]	70

Table 2. Data for calculating the cost of uncertainty

Once the process is completed, the programmed power graph $W_{W,s,i}$ is obtained with respect to the expected uncertainty cost, which is presented below. (Figure 4):

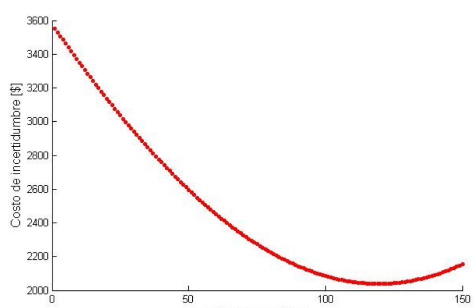


Figure 4. *Expected value of uncertainty cost for wind generation*

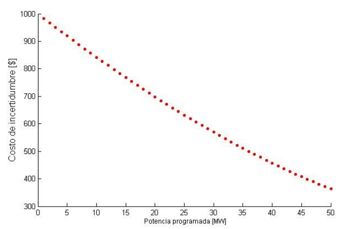
From this information, it is possible to achieve a modeling of the uncertainty cost function that has a polynomial structure taking as an independent variable the programmed power [3].

4.4. Polynomial and quadratic modeling of the uncertainty cost function

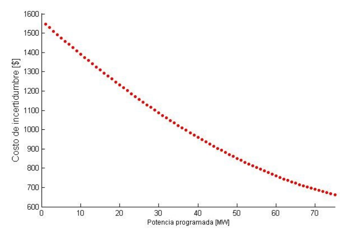
The approximate polynomial function that correctly represents the graph in Figure 4 does not have the form of the function 25, the classic form of the function is cost in the economic dispatch models. To this is added that it would not be possible to use the same methodology as the case of the photovoltaic generator (Section 3.4), since its behavior is mainly decreasing, so the domain of the new uncertainty cost function, which takes into account only the part whose slopes are positive, would be small; thus giving it a limited range of operation.

$$C_{Gi}(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i [P_{Gi}]^2 \tag{25}$$

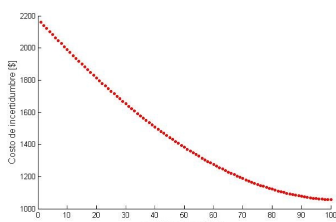
When performing the exercise with different nominal power values W_r, the behavior of the uncertainty cost function is similar, as observed in 5.



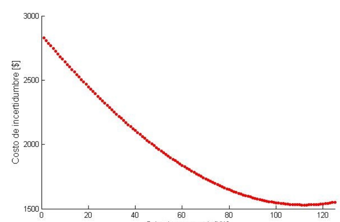
(a) $W_r = 50$



(b) $W_r = 75$



(c) $W_r = 100$



(d) $W_r = 125$

Figure 5. Behavior of the uncertainty cost function for several values of W_r

Therefore, the use of the income data presented in [14] is proposed, with a variation in the W_r. The data for the realization of this study are presented in the Table 3 [14].

When the Monte Carlo simulation is carried out again, the programmed power graph $W_{w,s,i}$ is obtained with respect to the expected uncertainty cost under the new conditions, which is presented below. on (Figure 6):

Income data		
Symbol	Parameter	Value
W_{wr}	Nominal power of the generator i [MW]	20
v_i	Boot wind speed $[m/s]$	5
v_r	Nominal wind speed $[m/s]$	15
v_o	Stop wind speed $[m/s]$	25
ρ	Linear constant of the power vs. speed curve $[MW/m/s]$	2
κ	Independent constant of the power vs. speed curve $[MW]$	-10
σ	Scale parameter of the Rayleigh distribution[m/s]	9
N	Number of iterations	1000000
$W_{w,s,i}$	Power programmed in the generator i [MW]	
$C_{w,u,i}$	Penalty cost coefficient due to underestimation [$/MWh]	30
$C_{w,o,i}$	Penalty cost coefficient due to overestimation [$/MWh]	70

Table 3. Data for calculating the cost of uncertainty

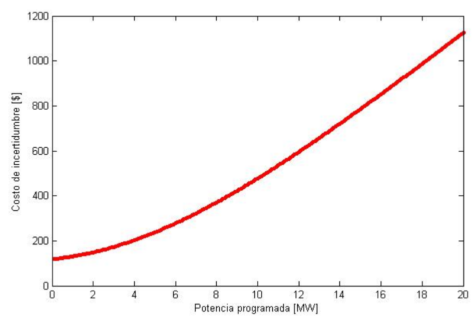


Figure 6. *Expected value of the cost of uncertainty for the generation of 20 MW with the conditions of the table 3*

From this information, it is possible to achieve a modeling of the uncertainty cost function that has a polynomial structure taking as an independent variable the programmed power and its quadratic approximation (Equation 26), with its respective comparison (Figure 7):

$$f(W_{w,s,i}) = 1.744(W_{w,s,i})^2 + 3.643(W_{w,s,i}) + 183.851 \tag{26}$$

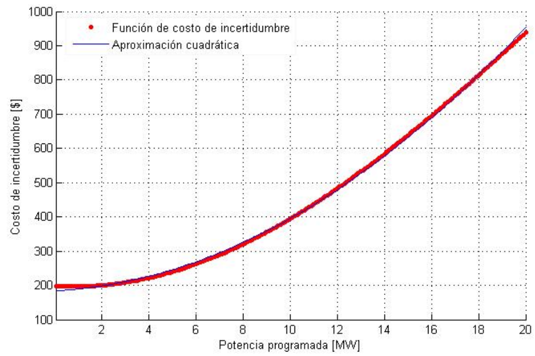


Figure 7. *Quadratic approximation of the cost of uncertainty for wind generation*

5. Uncertainty cost function small hydroelectric power plants

For small hydroelectric power plants, the primary energy source is the river flow (Q) in which the power plant is located. It can be considered that its behavior follows the distribution function of Gumbel [7] [8] [9]. Thus, the Gumbell probability distribution is described by [**?**]:

$$f(Q) = \frac{e^{[\frac{Q-\mu}{\sigma}]} e^{-e^{[\frac{Q-\mu}{\sigma}]}}}{\sigma} \tag{27}$$

where Q is the flow of water received by the generator, μ is the average value and σ is the mean square deviation. For the use of this distribution.

Additionally, the relationship between the water flow and the active power generated by a hydraulic generator is given by[7][8][9]:

$$W_H = 9.81 \cdot \rho \cdot \eta_t \cdot \eta_g \cdot \eta_m \cdot Q \cdot h \tag{28}$$

where W_H is the generated active power, ρ, is the water density in [kg / m^3], η_t is the efficiency of the Hydraulic turbine, eta_g is the efficiency of the generator, η_m is the efficiency of the coupling between the turbine and the generator, Q is the flow rate and h is the height difference in the central [m].

Under these considerations, [10] presents in detail the mathematical development of the cost function of uncertainty, both due to overestimation and underestimation.

5.1. Mathematical formulation in the underestimated condition

The formulation of the cost of uncertainty due to underestimation is presented as follows:

$$E[C_{H,u,i}(W_{Hi}, W_{H,i})] = \int_{W_{H,s,i}}^{W_{H,\infty,i}} c_{H,u,i}(W_{H,i} - W_{H,s,i}) \cdot fw_H(W_{H,i}) \cdot dW_{H,i} \tag{29}$$

where $E[C_{H,u,i}(W_{H,s,i}, W_{H,i})]$ is the expected value of the cost due to the underestimation, $c_{H,u,i}$ is the penalty cost coefficient for underestimation, $fw_H(W_{H,i})$ is the probability distribution of the primary source, $W_{H,s,i}$ is the power programmed by the economic dispatch model and $W_{H,\infty,i}$ is the maximum possible power in the generator [3].

Finally, the expression for the expected cost due to the underestimation is described below. [10]:

$$
\begin{aligned}
E[C_{H,u,i}(W_{H,s,i},W_{H,i})] = c_{H,u,i}\Bigg\{ & k\sigma\left[-e^{-e^{[\frac{(\frac{W_{H,\infty,i}}{k})-\mu}{\sigma}]}}\ln\left(e^{\frac{(\frac{W_{H,\infty,i}}{k})-\mu}{\sigma}}\right)\right. \\
& \left. +Ei\left(-e^{\frac{(\frac{W_{H,\infty,i}}{k})-\mu}{\sigma}}\right)\right] + (\mu k - W_{H,s,i})\left[-e^{-e^{\frac{(\frac{W_{H,s,i}}{k})-\mu}{\sigma}}}\right]\Bigg\} \\
-c_{H,u,i}\Bigg\{ & k\sigma\left[-e^{-e^{\frac{(\frac{W_{H,s,i}}{k})-\mu}{\sigma}}} ln\left(e^{\frac{(\frac{W_{H,s,i}}{k})-\mu}{\sigma}}\right) + Ei\left(-e^{\frac{(\frac{W_{H,s,i}}{k})-\mu}{\sigma}}\right)\right] \\
& +(\mu k - W_{H,s,i})\left[-e^{-e^{\frac{(\frac{W_{H,s,i}}{k})-\mu}{\sigma}}}\right]\Bigg\}
\end{aligned}
\tag{30}
$$

5.2. Mathematical formulation in the overestimated condition

The formulation of the cost of uncertainty due to overestimation is presented as follows:

$$E[C_{H,o,i}(W_{H,i},W_{H,i})] = \int_{W_{H,s,i}}^{W_{H,\infty,i}} c_{H,o,i}(W_{H,i} - W_{H,s,i}) \cdot fw_H(W_{H,i}) \cdot dW_{H,i} \tag{31}$$

where $E[C_{H,o,i}(W_{H,s,i},W_{H,i})]$ is the expected value of the cost due to overestimation, $c_{H,o,i}$ is the penalty cost coefficient for overestimation, $fw_H(W_{H,i})$ is the probability distribution of the primary source, $W_{H,s,i}$ is the power programmed by the economic dispatch model and $W_{H,\infty,i}$ is the maximum possible power in the generator.

Finally, the expression for the expected cost due to the overestimation is described below. [10]:

$$
\begin{aligned}
E[C_{H,o,i}(W_{H,s,i},W_{H,i})] = c_{H,o,i}\Bigg\{ & (W_{H,s,i} - \mu k)\left[-e^{-e^{\frac{(\frac{W_{H,s,i}}{k})-\mu}{\sigma}}}\right] \\
& -k\sigma\left[-e^{-e^{\frac{(\frac{W_{H,s,i}}{k})-\mu}{\sigma}}} ln\left(e^{\frac{(\frac{W_{H,s,i}}{k})-\mu}{\sigma}}\right) + Ei\left(-e^{\frac{(\frac{W_{H,s,i}}{k})-\mu}{\sigma}}\right)\right]\Bigg\} \\
-c_{H,o,i}\Bigg\{ & (W_{H,s,i} - \mu k)\left[-e^{-e^{\frac{(\frac{0}{k})-\mu}{\sigma}}}\right] \\
& -k\sigma\left[-e^{-e^{\frac{(\frac{0}{k})-\mu}{\sigma}}} ln\left(e^{\frac{(\frac{0}{k})-\mu}{\sigma}}\right) + Ei\left(-e^{\frac{(\frac{0}{k})-\mu}{\sigma}}\right)\right]\Bigg\}
\end{aligned}
\tag{32}
$$

Therefore, the uncertainty cost function for the energetic power plant is the result of the sum of the equations 30 and 32.

5.3. Monte Carlo simulation

Following the step of subsection 3.3, it is possible to obtain the respective Monte Carlo simulation. The data for the realization of this study are presented in the Table 4 [10].

Income data		
Symbol	Parameter	Value
W_H	Maximum power of the generator i [MW]	10
ρ	Water density $[kg/m^3]$	1000
η_t	Efficiency of the hydraulic turbine	0.9
η_g	Efficiency of the electric generator	0.95
η_m	Efficiency of the turbine-generator mechanical coupling	0.98
h	Height of the dam $[m]$	20
μ	Average value of the flow$[m^3/s]$	15.23
σ	Average deviation of the flow$[m^3/s]$	1.15
N	Iteration number	1000000
$W_{H,s,i}$	Power programmed for the generator i [MW]	
$C_{H,u,i}$	Penalty cost coefficient due to underestimation [$/MWh]	30
$C_{H,o,i}$	Penalty cost coefficient due to overestimation [$/MWh]	70

Table 4. Data for calculating the cost of uncertainty

Once the process is completed, the programmed power graph $W_{H,s,i}$ is obtained with respect to the expected uncertainty cost, which is presented below:

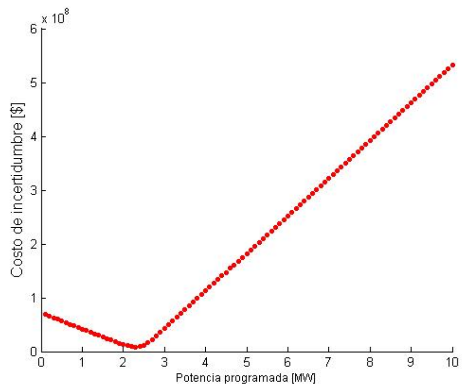


Figure 8. *Expected value of the cost of uncertainty for small hydroelectric power plants*

From this information, it is possible to achieve a modeling of the uncertainty cost function that has a polynomial structure taking as an independent variable the programmed power [3].

5.4. Modeling the uncertainty cost function

Through the use of numerical method tools, it is possible to obtain a polynomial function that describes in a much simpler way the behavior of the expected value of the cost of uncertainty, that allows its inclusion in simulators of optimization of power systems without having to resort to expressions such as those described in the equations 30 o 32.

In this case, the degree of polynomial with which a good approximation was achieved was grade 8. This polynomial is presented below (Figure 9), with the characteristic function (Equation 33):

$$\begin{aligned} f(W_{PV,s,i}) &= 0.0626(W_{H,s,i})^8 - 1.235(W_{H,s,i})^7 - 9.507(W_{H,s,i})^6 \\ &+452.295(W_{H,s,i})^5 - 4779.257(W_{H,s,i})^4 + 22324.884(W_{H,s,i})^3 \\ &-41808.858(W_{H,s,i})^2 + 518192.288(W_{H,s,i}) + 19352.597 \end{aligned} \quad (33)$$

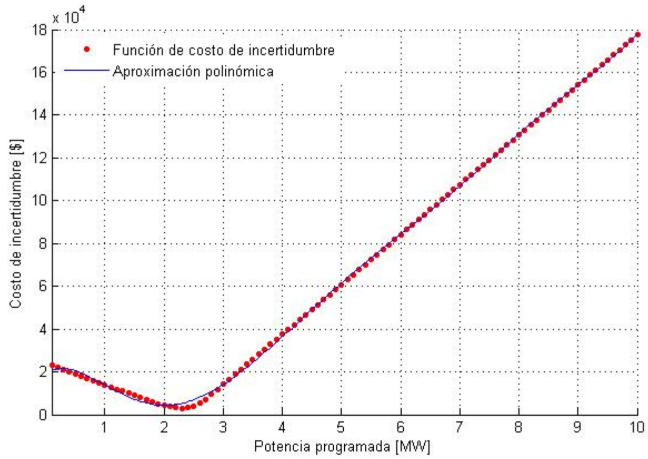


Figure 9. *Polynomial approximation of the cost of uncertainty for small hydroelectric power station*

This function largely represents the behavior of the expected value of uncertainty cost, however, it does not have the form of the function 34 normally used in the economic dispatch models. This is why, following the methodology of the section 3.4, the scenario in which this plant has a power value dispatched $W_{H,s,i}$ minimum, on in this way, the cost function of uncertainty can be described by a quadratic function, and thus be included in the economic dispatch models. [3]:

$$C_{Gi}(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i [P_{Gi}]^2 \quad (34)$$

Therefore, the quadratic function that achieves the best approximation to this new condition of the uncertainty cost function (Equation 35) with its corresponding graphic representation (Figure 10).

$$\begin{aligned} f(W_{PV,s,i}) = 58.194(W_{PV,s,i})^2 + 22520.517(W_{PV,s,i}) \\ -53241.786 \end{aligned} \quad (35)$$

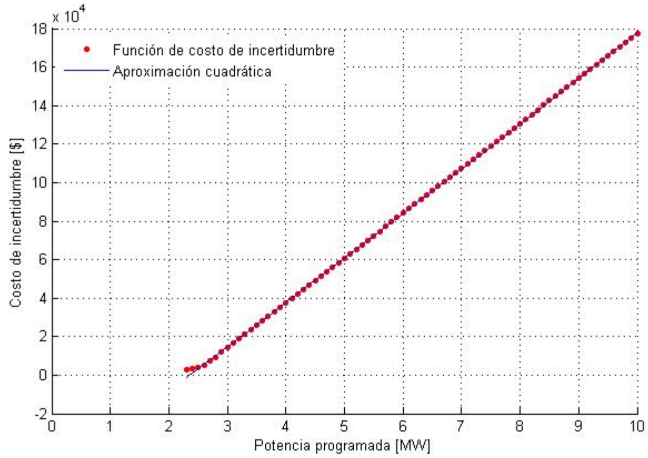


Figure 10. *Quadratic approximation of the cost of uncertainty for a small hydroelectric power station*

For inclusion in the economic dispatch models, this function of uncertainty costs must be accompanied by the condition 36:

$$W_{H,s,i} \geq 2.3MW. \tag{36}$$

6. Application of uncertainty cost functions in economic dispatch and discussion models

Once the quadratic cost functions of uncertainty are available, it is possible to include them in economic dispatch models. In this the distribution of the demand of the system among all the available generators is carried out, in such a way that the lowest possible cost is obtained for the generation of energy. This concept becomes a problem of optimization of the magnitude (V_m) and angle (Θ) of the tensions; and the active power injections (P_g) and reactive (Q_g) [15].

Thus, the objective function can be described as the sum of the individual cost polynomial functions for both active and reactive power injections. This function is described below [15]:

$$\min_{\Theta,V_m,P_g,Q_g} \sum_{i=1}^{n_g} f_P^i(p_g^i) + f_Q^i(q_g^i) \tag{37}$$

Also, this objective function is subject to equality and inequality equations. The equality constraints are not more than the nonlinear equations of active and reactive power balance (Equation 38). Regarding the inequality constraints, they consist of two sets of branch flow lites as nonlinear functions of the node voltage angles and magnitudes, one for each branch end. (*from end* and *from end* - Equation 39) [15]:

$$\begin{aligned} g_P(\Theta, V_m, P_g) &= P_{bus}(\Theta, V_m) + P_d - C_g P_g = 0 \\ g_Q(\Theta, V_m, P_Q) &= Q_{bus}(\Theta, V_m) + Q_d - C_g Q_g = 0 \end{aligned} \tag{38}$$

$$h_f(\Theta, V_m) = |F_f(\Theta, V_m)| - F_{max} \leq 0$$
$$h_t(\Theta, V_m) = |F_t(\Theta, V_m)| - F_{max} \leq 0 \quad (39)$$

There are informatics aids that allow the programming of economic dispatch models, and, also, solve the optimization problem that we face. One of these aids is the MATPOWER tool, designed to solve power flow problems and optimal power flow problems; It is especially aimed at researchers and educators [3].

In this section, the results of four optimal power flow simulations will be presented, in which the uncertainty cost functions, obtained throughout this study, are implemented in the "CASE 9" ; a power system of nine nodes and three generators, containing specific charges and lines.

6.1. Case 1: Power system with solar generation

The photovoltaic generator is entered with the information of the Table 5.

Generator maximum power	$70MW$
Generator nominal power	65 MW
Generator minimum power	25 MW
Cost function type	Polynomial of 2^{do} degree
Cost function	See equation 13
Location	Node 3

Table 5. Input data to the optimal power flow

6.1.1. Economic dispatch without losses by transmission

After performing the economic dispatch exercise without transmission losses, the results recorded in the Table are obtained 6:

Generator	Active power dispatched [MW]
G. N°1	166.77
G. N°2	98.167
G. Solar	50.067
Total	315.004

Table 6. Case 1: Results of the economic dispatch without loss of transmission

with a final cost of 17180.26209 $/ hr.

6.1.2. Economic dispatch with losses by transmission

Thus, the results of the optimal power flow executed are presented in the Figure 11.

Bus Data

Bus #	Voltage Mag(pu)	Voltage Ang(deg)	Generation P (MW)	Generation Q (MVAr)	Load P (MW)	Load Q (MVAr)	Lambda($/MVA-hr) P	Lambda($/MVA-hr) Q
1	1.100	0.000*	167.11	12.50	-	-	66.765	-
2	1.097	-2.966	99.37	-2.04	-	-	66.893	-
3	1.088	-5.135	50.89	-20.73	-	-	67.233	-
4	1.097	-4.576	-	-	-	-	66.776	0.134
5	1.087	-7.852	-	-	90.00	30.00	68.135	0.152
6	1.100	-6.562	-	-	-	-	67.233	-
7	1.089	-8.161	-	-	100.00	35.00	67.599	0.100
8	1.100	-5.916	-	-	-	-	66.893	-
9	1.076	-8.325	-	-	125.00	50.00	67.960	0.391
		Total:	317.38	-10.26	315.00	115.00		

Figure 11. Optimal power flow results Case 1

With a final cost of 17339.17 $\$/hr$.

The comparison of cost functions of the default generators of the "Case 9" and the cost function of the photovoltaic generator can be clearly seen in the Figure 12.

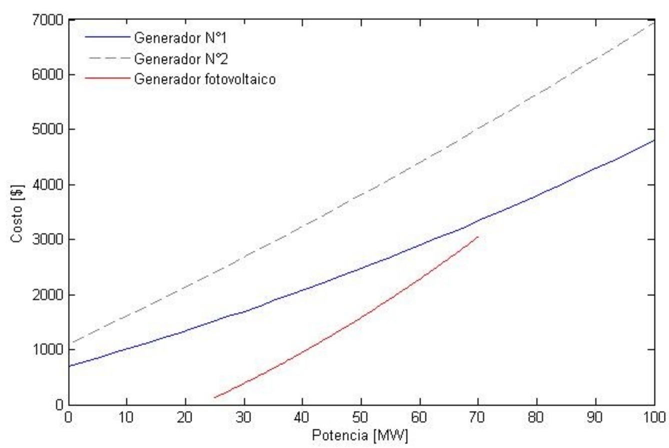


Figure 12. Comparison of cost functions in the optimal power flow Case 1

6.2. Case 2: Power system with wind generation

The wind generator is entered with the information of the Table 7.

Generator maximum power	20 MW
Generator nominal power	20 MW
Generator minimum power	0 MW
Cost function type	Polynomial of 2^{do} degree
Cost function	See equation 26
Location	Node 3

Table 7. Input data to the optimal power flow

6.2.1. Economic dispatch without losses by transmission

After performing the economic dispatch exercises without loss of transmission, the results recorded in the table are obtained 8.

Generator	Active power dispatched [MW]
G. N^o1	180.34
G. N^o2	115.73
G. Wine	18.931
Total	315.001

Table 8. Case 2: Results of the economic dispatch without loss of transmission

With a final cost of 18590.38501 $/hr.

6.2.2. Economic dispatch with losses by transmission

Thus, the results of the optimal power flow executed are presented in the Figure 13.

Bus Data

Bus #	Voltage Mag(pu)	Voltage Ang(deg)	Generation P (MW)	Generation Q (MVAr)	Load P (MW)	Load Q (MVAr)	Lambda($/MVA-hr) P	Lambda($/MVA-hr) Q
1	1.100	0.000*	180.58	14.76	-	-	69.728	-
2	1.099	-2.873	117.83	2.30	-	-	70.031	-
3	1.084	-8.021	19.29	-21.32	-	-	70.937	-
4	1.096	-4.948	-	-	-	-	69.742	0.164
5	1.085	-8.803	-	-	90.00	30.00	71.386	0.193
6	1.095	-8.567	-	-	-	-	70.937	-
7	1.087	-9.263	-	-	100.00	35.00	71.002	0.114
8	1.100	-6.364	-	-	-	-	70.031	0.000
9	1.075	-8.725	-	-	125.00	50.00	71.037	0.431
		Total:	317.70	-4.26	315.00	115.00		

Figure 13. Optimal power flow results Case 2

with a final cost of 18779.39 $/hr.

The economic clearance exercise can be clearly seen in Figure 14, where the comparison of the cost curves of the default generators of ”Case 9” and the cost curve are observed of the generator [3].

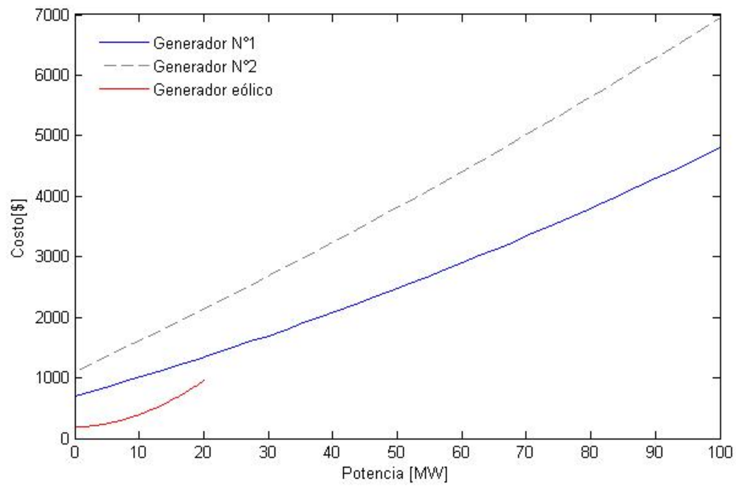


Figure 14. Comparison of cost functions in the optimal power flow Case 2

6.3. Case 3: Power system with small hydroelectric

The generator of the small hydroelectric power station is entered with the information of the Table 9.

Generator maximum power	10 MW
Generator nominal power	10 MW
Generator minimum power	2.3 MW
Cost function type	Polynomial of 2^{do} degree
Cost function	See equation35
Location	Node 3

Table 9. Optimal power flow results Case 3

6.3.1. Economic dispatch without losses by transmission

After performing the economic dispatch exercise without transmission losses, the results recorded in the Table are obtained 10.

Generator	Active power dispatched [MW]
G. N^{o}1	187.59
G. N^{o}2	125.11
PCH	2.3
Total	315

Table 10. Case 3: Results of the economic dispatch without loss of transmission

with a final cost of 17747.81378 $/hr.

6.3.2. Economic dispatch with losses by transmission

Thus, the results of the optimal power flow executed are presented in the Figure 15.

Bus Data

Bus #	Voltage Mag(pu)	Voltage Ang(deg)	Generation P (MW)	Generation Q (MVAr)	Load P (MW)	Load Q (MVAr)	Lambda($/MVA-hr) P	Lambda($/MVA-hr) Q
1	1.100	0.000*	187.85	15.80	-	-	71.328	-
2	1.100	-2.818	127.84	4.22	-	-	71.733	-
3	1.083	-9.595	2.30	-19.82	-	-	72.974	-
4	1.096	-5.149	-	-	-	-	71.345	0.191
5	1.085	-9.318	-	-	90.00	30.00	73.153	0.227
6	1.094	-9.661	-	-	-	-	72.974	-
7	1.086	-9.860	-	-	100.00	35.00	72.862	0.139
8	1.100	-6.605	-	-	-	-	71.735	0.024
9	1.075	-8.940	-	-	125.00	50.00	72.705	0.469
		Total:	318.00	0.20	315.00	115.00		

Figure 15. Optimal power flow results Case 3

with a final cost of 17962.03 $/hr.

In these results the dispatch of the maximum power can be observed, this due to the clear differences between the cost in the generators 1 and 2 with respect to the generator of the small hydroelectric power station, as it can be see in the Figure 16.

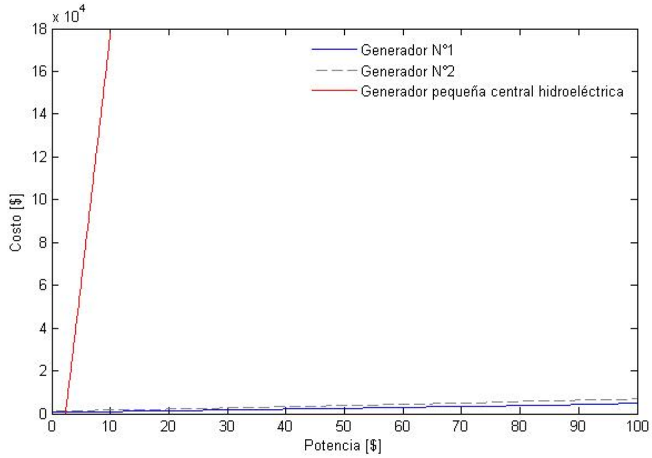


Figure 16. Comparison of cost functions in the optimal power flow Case 3

6.4. Case 4: Power system with combination of technologies

In this case, the photovoltaic generator (see Table 5) is taken into account in node 2 and the generator (see Table 7) in node 3, which presented better behavior to the be included in the power system model of "Case 9". As for the small hydroelectric power generation plant, described throughout this article, it does not present a cost function that is competitive with the other generators. used in power flows.

6.4.1. Economic dispatch without losses by transmission

After performing the economic dispatch exercise without transmission losses, the results recorded in the Table are obtained 11

Generator	Active power dispatched [MW]
G. N^o1	225.44
G. Solar	69.565
G. Wind	20
Total	315.005

Table 11. Case 4: Results of the economic dispatch without loss of transmission

with a final cost of 17024.39941 $/hr.

6.4.2. Economic dispatch with losses by transmission

The response of the power flow to the new conditions is presented in the Figure 17

Bus Data

Bus	Voltage		Generation		Load		Lambda ($/MVA-hr)	
#	Mag(pu)	Ang(deg)	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)	P	Q
1	1.100	0.000*	228.45	23.07	-	-	80.258	-
2	1.097	-9.316	70.00	2.79	-	-	82.479	-
3	1.081	-11.573	20.00	-18.95	-	-	83.046	-
4	1.094	-6.275	-	-	-	-	80.294	0.326
5	1.082	-10.926	-	-	90.00	30.00	82.638	0.374
6	1.092	-12.142	-	-	-	-	83.046	-
7	1.083	-13.706	-	-	100.00	35.00	83.418	0.127
8	1.096	-11.402	-	-	-	-	82.479	0.000
9	1.073	-11.340	-	-	125.00	50.00	82.274	0.666
		Total:	318.45	6.91	315.00	115.00		

Figure 17. Optimum power flow results Case 4

With a final cost of 17299.82 $\$/hr$.

By observing the power flow response in cases 1 and 2 (Figures 11 and 13) it is possible to infer a dispatch at maximum power for this case, given that a decrease occurs. Installed capacity added to the dispatch trend near the maximum in the case 2 13 and of a 71% of the installed capacity of the photovoltaic generator in case 1 (Figure 11).

Likewise, the economic clearance exercise can be clearly seen in Figure 18, where the comparison of the cost curves of one of the predetermined generators of the "Case 9" and the curves of the generators wind and solar [3].

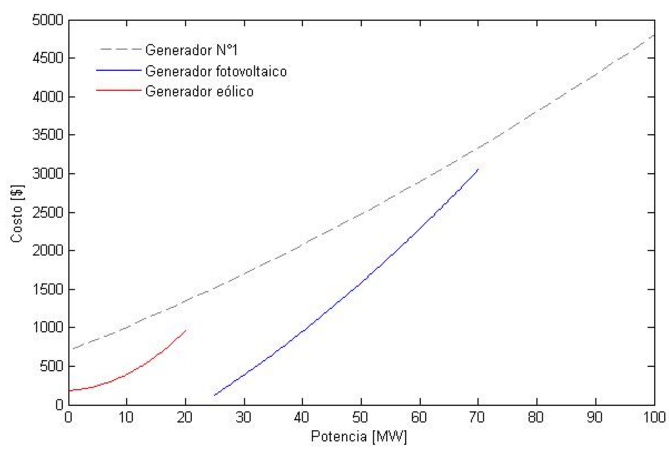


Figure 18. Comparison of cost functions in the optimal power flow Case 4

7. Conclusions

The quadratic approximation of the costs of uncertainty for unconventional renewable sources allows its inclusion in economic dispatch models and thus take an important step in economic issues. who are more concerned when evaluating the viability of these projects. Likewise, it can be deduced that the correct determination of the stochastic variables (wind speed, radiation and flow), allows a better approximation to the functions of uncertainty costs and, to its time, a better economic clearance exercise.

As expected, the uncertainty cost functions obtained are valid only for each case and vary according to the specific conditions of each plant, so this study, and others like [6], [10] or [14], constitute a methodological basis for future renewable generation projects.

On the other hand, it is important to note that the results obtained in the power flows are linked to the conditions of the predetermined network in the MATPOWER program, and, therefore, the behavior or trend of scheduled dispatch will be subject. to the actual conditions of the loads, branches, and other elements of a power system.

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