Universidad del Atlántico

MATUA

Revista Del Programa De Matemáticas Páginas: 29–35

Facultad de Ciencias Básicas

Vol. V , Nº 1, (2018)

Theorem of the real numerical value of a polynomial according to the derivatives of higher order

Teorema del valor numérico real de un polinomio en función a las derivadas de orden superior .

Brandon Smith Martínez Costa.

Universidad de Pamplona-Colombia.

brandm7945@gmail.com

Abstract

In this paper, we will test the real numerical value of a polynomial function of the form $y = f(x)$ of degree n; by the expression: $f(x) = \frac{d^n y}{dx^n}$ such that, $x \in \mathbb{R}$ for all x positive and negative. In the present work, the applications of the real numerical value to simple measurements of the geometry are studied, making use of the derivatives of higher order.

Keywords: Real numerical value, derived from higher order, mathematical equality, geometry, perimeter.

Resumen

En este artículo, se probara el valor numérico real de una función polinómica de la forma $y = f(x)$ de grado n; mediante la expresión: $f(x) = \dfrac{d^n y}{dx^n}$ tal que, $x \in \mathbb{R}$ para todo x positivo y negativo. En el presente trabajo, se estudian las aplicaciones del valor numérico real a medidas simples de la geometría, haciendo uso de las derivadas de orden superior.

Palabras claves: Valor numérico real, derivada de orden superior, igualdad matemática, geometría, perímetro.

1. Introducción

A polynomial function is one that consists of several algebraic terms, these functions can be of degree n [1]. In general, expressions such as $p(x) = a_0 + a_1 x + a_2 x^2 + ?a_3 x^3 + \cdots + a_n x^n$, denote as a polynomial function of degree n, being $a_n \neq 0$. It is important to highlight that, in the present study, functions will work in a real variable $y = f(x)$. On the other hand, the numerical value of a polynomial is the value which takes when the variables are replaced by certain values, however, for the actual value of a function we use the concept of derivative of higher order, applied to the calculation and has basic examples geometry. If f is derivable [2], then its derivative f' is also a new function, which is known as one for higher order.

Definition 1.1. If $y = f(x)$ is a function derivable, then it's for $f'(x)$ is also a function, so $f'(x)$ may have a derivative of if same, designated by $(f')' = f''$. This new function f'' is known as the second derivative of $f(x)$.

$$\frac{d}{dx}(dy/dx) = \frac{d^2y}{dx^2} \qquad (1)$$

Theorem 1.2. It is $y = f(x)$, a polynomial function of degree n derivable, then the real numerical value is given by the expression:

$$f(x) = \frac{d^ny}{dx^n} \qquad (2)$$

Where: $\frac{d^ny}{dx^n}$ corresponds to the n-th derived from function and $f(x)$ corresponds to the function. Therefore, the value of the function x belongs to all the real:

$$\frac{d^ny}{dx^n} = \{x \in \mathbb{R}\}$$

Proof 1.3. Again be $y = f(x)$ a function derivable from grade n, $p : \mathbb{R} \to \mathbb{R}$ then, the real value of the function is given by the expression:

$$f(x) = \frac{dy}{dx} = \frac{d^2y}{dx^2} = \frac{d^3y}{dx^3} = \frac{d^ny}{dx^n} \qquad (3)$$

Then

$$\frac{d^ny}{dx^n} = \text{Real Value of the Function} \qquad (3,1)$$

Example 1.4. For the polynomial function $p(x) = -2x^4 - 5x^3 + 7x^2 - 9x + 6$ calculate:

I. The real value of $p(x)$ using the theorem 1.2
II. The values in x that satisfy the function $p(x)$

Applying the theorem 1.2 we know that $p(x)$ is a function of 4th grade.

$$p(x) = -2x^4 - 5x^3 + 7x^2 - 9x + 6$$

Then

$$p(x) = \frac{d^4y}{dx^4}$$

Calculating their derivatives must be:
$p'(x) = -8x^3 - 15x^2 + 14x - 9$
$p''(x) = -24x^2 - 30x + 14$
$p'''(x) = -48x - 30$
$p''''(x) = -48$

Therefore, $p''''(x) = -48$ corresponds to the actual numeric value of the function in this example.

Now, use the equation (3) for part II corresponding to 1.3 demonstration, with the objective of determining the values that satisfy the polynomial function $p''''(x) = -48$.

Knowing that:

$$y = p(x) = \frac{dy}{dx} = \frac{d^2y}{dx^2} = \frac{d^3y}{dx^3} = \frac{d^4y}{dx^4}$$

We determine the values for each derived through the expression (3):

$$\frac{d^3y}{dx^3} = \frac{d^4y}{dx^4} \qquad (1,0)$$

$$-48x - 30 = -48 \qquad (1,1)$$

By solving the equation (1.1) we have:

$$-48x + 18 = 0$$

$$x = 0,375$$

Checking the mathematical equality (1.0) must be:

$$-48(0,375) - 30 = -48 - 48 = -48$$

Then

$$p'''(x) = -48x - 30$$

$$p'''(0,375) = -48(0,375) - 30 = -48$$

For the second derivative, we carry out the same treatment:

$$\frac{d^2y}{dx^2} = \frac{d^3y}{dx^3} \qquad (1,2)$$

$$-24x^2 - 30x + 14 = -48 \qquad (1,3)$$

Note 1.1. For the second derivative the linear expression $-30x$, takes the value of $x = 0,375$; by which, it is required to determine the value of x^2 corresponding to the term $-24x^2$.

Solving the expression (1.3) must be:

$$-24x^2 + 50{,}75 = 0$$

$$x = \sqrt{\frac{50,75}{24}} = 1,4541$$

Here are the values of x that satisfy the real value of the function $p(x) = -48$

With values in their derivative of:

$p(2,3165) = -?2x?^4 - 5x^3 + 7x^2 - 9x + 6$

$p'(1,1614) = -8x^3 - 15x^2 + 14x - 9$

$p''(1,4541) = -24x^2 - 30x + 14$
$p'''(0,375) = -48x - 30$
$p''''(x) = -48$
Finally, the values of x that satisfy the actual numeric value of the polynomial are taken.
$p(2,31;1,16;1,45;0,375) = -2x^4 - 5x^3 + 7x^2 - 9x + 6 = -48$
$p(x) = -2(2{,}31)^4 - 5(1,16)^3 + 7(1,45)^2 - 9(0,375) + 6 = -47,40 \cong -48$

Note 1.2. For this example, all belonging to x decimal places were not taken.

Lemma 1.5. The reason for change $\frac{dv}{dt}$, which displays the name of acceleration, denote it by means of a, then [3]:

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \qquad (4)$$

We know the expression (4) as a physical application of the higher-order derivatives. However, this expression does not lead to the real function numeric value, since they are reasons for change applications.

2. Applications to geometry

The analysis of the real numeric value studied, is proposed measures as the perimeter in function of the area $P(A)$, for two plane figures. For this reason, the area of a figure is interpreted as a function $y = f(u)$, where u corresponds to figure square units.

Definition 2.1. Knowing that you for a square of side, Z then its area is:

$$A = Z \times Z = Z^2$$

Theorem 2.2. For a square with vertices $ABCD$, its perimeter in terms of the area $P(A)$, is given by the expression:

$$P(A) = \sqrt{\left(\frac{dy}{du}\right)\times 2} + \sqrt{\left(\frac{d^2y}{du^2}\right)\times 2} \qquad (5)$$

Then, the area of a square with equal sides is:

$$f(u) = \frac{dy}{du} = \frac{d^2y}{du^2} \qquad (5,1)$$

Example 2.3. Is a square ABCD with sides equal of 7 cm. determine:
I. The perimeter according to your area using the actual numeric value theorem. (See Theorem 1.2)

Using the definition 2.1

$$A = 7^2 = 49cm^2$$

Interpreting the area asa square function and $y = f(u)$, is to:

$$f(u) = 49u^2$$

By Theorem 1.2 we determine the real numeric value of the function:
$f(u) = 49u^2$
$f'(u) = 98u$
$f''(u) = 98$
Then the real numeric value of the function corresponds to $f''(u) = 98$

Applying the theorem 1.2

$$\frac{d^2y}{du^2} = \frac{dy}{du}$$

$$98 = 98u$$

Therefore

$$u = 1$$

Now, by applying the theorem 2.2 is that the perimeter is:

$$P(A) = \sqrt{\left(\frac{dy}{du}\right)\times 2} + \sqrt{\left(\frac{d^2y}{du^2}\right)\times 2}$$

$$P(A) = \sqrt{98\times 2} + \sqrt{98\times 2}$$

$$P(A) = 28u$$

3. Estimation of the perimeter in the area $P(A)_R$ a rectangle function

This section proposes a new theorem for a rectangle, with the objective of determining its perimeter. It should be noted that $P(A)_R$ is an exact value, while $P(A)$ is an approximation of the perimeter.

Theorem 3.1. Suppose a rectangle ABCD, with base b and height h. Then its perimeter in terms of area $P(A)_R$, is denoted as:

$$P(A)_R = \left(\frac{A}{b}\times 2\right) + \left(\frac{A}{h}\times 2\right) \qquad (6)$$

Therefore: $P(A)_R \in \mathbb{R}$

Proof 3.2. Knowing that the area and the perimeter for a rectangle is:

$$A = b\times h \qquad (6,1)$$

$$P = 2b + 2h \qquad (6,2)$$

Solving b and h of the expression (6.1)

$$b = \frac{A}{h} \qquad (6,3); \quad h = \frac{A}{b} \qquad (6,4)$$

Now, replacing (6.3) and (6.4) in the expression (6.2)shows that:

$$P(A)_R = \left(\frac{A}{b}\times 2\right) + \left(\frac{A}{h}\times 2\right)$$

Example 3.3. For a rectangle of 5 cm from base, and a height of 7 cm. Determine the following:

I. Perimeter in function of the area $P(A)_R$

II. The real numeric value of the area function.

III. An estimate of the perimeter by means of Theorem 1.2

I. To this point, is known by definition that the area of a rectangle is given by:

$$A = b\times h$$

Then

$$A = 5\times 7 = 35\ cm^2$$

Applying the theorem 3.1

$P(A)_R = \left(\frac{35cm^2}{5cm}\times 2\right) + \left(\frac{35cm^2}{7cm}\times 2\right)$

$P(A)_R = 24cm$

II.Now, interpreting the area as a function of the square and deriving the function is:

$f(u) = 35u^2$
$f'(u) = 70u$
$f''(u) = 70$

Then the real numeric value of the function is $f''(u) = 70$

III. The estimation of the perimeter, this effected with the theorem of the real numerical value (see Theorem 1.2). This perimeter is an approximation to the exact.

$P(A) = \sqrt{\left(\frac{dy}{du}\right)\times 2} + \sqrt{\left(\frac{d^2y}{du^2}\right)\times 2}$

$P(A) \cong \sqrt{70\times 2} + \sqrt{70\times 2}$

$P(A) \cong 23,66u(units)$

Analyzing a percentage of error, it is that:

$\%E = \frac{RealPerimeter - ApproximatePerimeter}{RealPerimeter}\times 100$

$$\%E = \frac{24 - 23,66}{24} \times 100 = 1,41\,\%$$

Thanks

To God and my parents.

Referencias

[1] Hernando L. Leal, differential calculus in a real variable, Universidad Popular de Cesar-UPC, 2008.

[2] J. Stewart, calculus of one variable. Transcendent early, 7aEd. Mexico: CENGAGE Learning, 2012.

[3] Purcell, Edwin J.; Varberg, Dale; Rigdon, Steven E, cálculo. Pearson Educación, México, 2007. Disponible en:
$https : //bibliotecavirtualmatematicasunicaes.files.wordpress.com\ /2011/11/cc3a1lculo_edwin - purcell - 9na - edicic3b3n.pdf$